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THE DYNAMICS OF LARGE-SCALE ATMOSPHERIC PROCESSES

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THE DYNAMICS OF LARGE-SCALE ATMOSPHERIC PROCESSES

(This analysis was made in 1951)

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Introduction

A complete conception of large-scale atmospheric process determining the nature of the weather in many respects, can only be obtained by the inclusion of all basic factors in the baroclynic atmosphere.

In the analysis of large-scale atmospheric processes we find it necessary to formulate mathematical theories which sh the nature and evolution of the real fields of meteorological elements as exactly as possible. In principle, the mathematic constructions based on the application of the laws of hydrothermodynamics in atmospheric conditions permit the pertinent system of nonlinear equations to determine the meteorological elements of interest to us. Therefore, in forecasting meteorological elements the solution of the complete system of nonlinear hydrothermodynamic equations appears to be the fundamen assignment for meteorological theory.

The attempt to obtain weather forecasts the theoretical way by studying the system of hydrodynamic equations was made for the first time by Richardson (9) in 1922. Richardson did not obtain sufficient results because he did not tackle the problem the right methodical way. Richardson took the initial wind field as the starting fact and hoped to obtain the evolution of atmospheric currents with time on the basis of the solution of the equation system. Inasmuch as the initial field of velocity generally includes manifold perturbations of all kinds of scales, Richardson should have taken into account the evolution of all types of small (sound, gravitational) perturbations together with the evolution of large-scale perturbations which are of meteorological interest, although the former are insignificant in meteorological respects. The description of the small perturbations require the utilization of correspondingly small scales and small time integrals. The length and time scales used by Richardson, were much larger than the degree of exactitude required; therefore, the sole forecast calculated by Richardson did not stand the test.

The first feasible solution of the equation system for atmospheric processes was found by I.A. Kibel (2) in 1940. He based his analysis on the assumption that large-scale atmospheric processes present themselves as quasi-geostrophic processes. This permitted I.A. Kibel to take the initial fields of pressure and temperature as starting factors and not the initial wind field; his problem was the calculation of changing pressure and temperature fields in time.

However, in I.A. Kibel's analysis the possibilities of qu geostrophic samples were not completely exhausted. Therefore, the temperature derivative of time $\frac{\partial T}{\partial t}$ was only determined wi a calculation of the horizontal mass transfer, and in the pres sure change $\frac{\partial p}{\partial t}$ the convergence factor and the divergence of the air in the atmosphere were not considered.

In other meteorological analyses of the Friedmann-Kotshin School, the system of hydrothermodynamic equations was used mo efficiently; the vortex velocity equation was examined as a pr moting equation connecting the complete alteration of the vortex with the divergence of the air on the horizontal plane.

In 1941 M.E. Shvetz (6) obtained the expression for the ve tical velocity in the shape of the integral according to altitude from the individual derivative of the vortex velocity.

In 1943 E.N. Blinova (1) managed to obtain the prognostic equations for pressure and temperature T on a certain "average" atmospheric level through linearization, taking into account th sphericity of the surface of the earth. E.N. Blinova started with the condition of maintenance of vorticity of PARCELS on the mean level and with the assumption that temperature changes in the atmosphere depend only upon the horizonta transfer of homogeneous masses of different temperatures. E.N. Blinova integrated the system of linear equations obtained in this manner, under general conditions of the elementary field

Foreign meteorologists see the possibility of utilizing hycrodynamic equations for weather forecasts in a different way.

N. Ertel (8) analysed the equation [1]

$$-\frac{\partial \Delta p}{\partial t} = \frac{1}{l\rho} (p, \Delta p)$$

obtained by him starting with the nature of the area of dependence of equation $\frac{\partial p}{\partial t}$ upon the field of function $(p, \Delta p)$; the result was a meteorologically incorrect conclusion according to which it is impossible to calculate the future pressure ma in advance. The deficiency was clearly reflected in the resul of A.M. Obuchov'd (3) analysis published in 1949 in which the author obtained a more complete equation in the examination of the barotrope sample of the atmosphere:

$$\frac{1}{L_1^2} \frac{\partial p}{\partial t} - \Delta \frac{\partial p}{\partial t} = \frac{1}{l\rho} (p, \Delta p),$$

$L_1 = \frac{\sqrt{gH_o}}{l}$; H_o - altitude of homogeneous atmosphere; the solution of this equation is

$$\frac{\partial p}{\partial t} = \int_0^\infty \frac{1}{l\rho} (p, \Delta p), K_o(r) r dr,$$

[1] We will keep to the symbols used in the first part of this analysis.

$$r = \frac{\sqrt{x^2 - y^2}}{L_1}, \quad K_o(r) - \text{Macdonald function of zero order.}$$

In 1950 N.I. Bulajev obtained the equation ~~[illegible]~~ for the change of pressure in the baroclynic atmosphere under the condition of a linear temperature decrease with height. The equation for the pressure change on the mean level of the troposphere was obtained as follows:

$$\frac{\partial p}{\partial t} - k_1 \Delta \frac{\partial p}{\partial t} = k_2(p, \Delta p) - k_3(\overline{T}, \Delta \overline{T}),$$

$\overline{T}$ - average temperature of troposphere; k_1, k_2, k_3 - a cert[ain] constant quantity.

This analysis also brought us the formula for the calculation of vertical velocity at different levels of the atmos[s]phere:

$$g\rho\omega = -a_1(z)\Delta(\overline{T}, p) - a_2(z)\left[(\overline{T}, \Delta p) + (p, \Delta\overline{T})\right],$$

p - pressure at the mean level of atmosphere; $a_1(z)$, $a_2(z)$ - the coefficient dependent on the altitude.

Charney's (7) analysis examined the barotropic sample o[f] the atmosphere and the simple baroclynic sample. The author uses the equation for the winds in geostrophic approximatio[n] and the equation of the heat flux without considering the v[er]tical current. Charney obtained Poisson's equation for the derivative of pressure at different levels. The resulting equation in the barotropic case was integrated according to the method of finite difference with the aid of electronic equipment. To integrate the equation, Charney was forced t supplement artificial boundary conditions, pressure values and Laplacian of pressure as time functions at the boundary of the area.

The baroclynic atmosphere sample was examined in the ana lysis published by I.A. Kibel (1953). The author started wit the equation of the vortex in geostrophical approximation a with the equation of the heat influx without considering the vertical current. I.A. Kibel integrated Poisson's equation for the desired function $\frac{\partial z}{t}$ on surfaces p = const and obtained the solution as follows:

$$\frac{\partial z}{\partial t} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{r_1} F(r, \phi, p) \ln \frac{r_1}{r} r dr d\phi - \frac{\overline{\partial z}}{\partial t},$$

$F(r, \phi, p)$ - the known function of the fields of meteorologi cal elements in the entire atmosphere, $\frac{\overline{\partial z}}{\partial t}$ - the mean value of the desired value at level p up to the circumference of radius r_1. Value $\frac{\partial z}{\partial t}$ with sufficiently large r_1 (1,000km) appears small and may be eliminated. Excluding the average value of function $F(r, \phi, p)$ under the integral symbol and integrating the function of influence $\frac{r_1}{r}$ I.A. Kibel obtained the prognostic formula

$$\frac{\partial z}{\partial t} = m\overline{F}(r, \phi, p),$$

$m = \frac{r_1^2}{4}$. For purposes of simplification it was assumed that function $\overline{F}(r, \phi, p)$ equals $F(r, \phi, p)$ in regard to the examined point.

In this analysis the authors intended to obtain general equations for pressure and temperature changes and an equation for the vertical currents which takes all basic factors of the baroclynic atmosphere into account. Compared to previous analyses the complete inclusion of dynamic factors in the temperature changes appears to be a new factor.

In the integration of the obtained differential equations the authors found solutions which were expressed by space integrals of the determined expressions, which depended upon the distribution of pressure fields and temperature in space as well as the congruent influence functions.

1. Formulation of Problem

In the study of the dynamics of pressure and temperature changes and the formation of vertical motion in the baroclyn atmosphere we used the system of hydrothermodynamic equation in regard to atmospheric processes on a large scale. A chara teristic feature of this system which distinguishes it from conventional hydrodynamic equations, appears to be the existence of the deflective force of the rotation of the earth in the equations of motion.

The local study of space regions with horizontal dimensions over several thousand kilometers permits us to consider the surface of the earth as a level surface subject to the limitations of the region under study and the use of the rectangular coordinate system in the initial equations.

The analysis excludes regions in the immediate proximity of the equator ($\phi = 0$-30°) because the nature of the motion shows fundamental changes in these regions.

The inital system of hydrodynamic equations is assumed as follows:

Equations of Motion

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + l v + \frac{1}{\rho}\frac{\partial}{\partial z}\mu\frac{\partial u}{\partial z}, \qquad (1.$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - l u + \frac{1}{\rho}\frac{\partial}{\partial z}\mu\frac{\partial v}{\partial z}, \qquad (1.$$

Equation of Statistics

$$\frac{\partial p}{\partial z} = -g\rho, \qquad ($$

Equation of Continuity

$$\frac{\partial \rho}{\partial t} - \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} - \frac{\partial \rho w}{\partial z} = 0, \qquad ($$

Equation of Heat Influx

$$\frac{dT}{dt} - \frac{\gamma_a}{g\rho}\frac{dp}{dt} = \frac{\varepsilon}{c_p},$$ (

Equation of State

$$p = \rho RT.$$ (

The following symbols were used in the equations (1.1)

(1.6):

u, v, w - components of velocity vector $\vec{V}$ to coordinate axis

p - pressure, ρ - density, T - air temperature, $l = 2\omega$co

ω - angular velocity of earth rotation, Θ - supplement to

cal latitude, μ - coefficient of turbulence, g - accelera

due to gravity, R - gas constant, ε - flux of heat per ma

unit dependent upon radiation and transition of water in at

mosphere, from one phase state to the other, c_p - specific

air under constant pressure, γ_a - adiabatic temperature gra

dient.

Boundary conditions and initial data are indispensible

the complete determination of the problem.

As boundary conditions with the earth surface we take t

transformation of the vertical velocity to zero

$$w = 0 \quad \text{with} \quad z = 0$$

and the conditions of the free surface at the upper limit of the atmosphere

$$\frac{dp}{dt} \to 0 \quad \text{with} \quad z \to \infty. \qquad (\ldots$$

Elementary data are pressure fields and temperatures at t =
p(x, y, z) and T(x, y, z).

It is advisable to go over to the coordinate system x', y', p, t', where pressure p is assumed to be an independent variable; axes x' and y' are on the isobaric surfaces. The a- titude of the isobaric surfaces z is now considered a functi- of coordinates x', y', p and time t'.

The transition from the elementary system of the coordi- x, y, z, t to the new system z', y', p, t' is realized thro- the following changes:

$$\left.\begin{aligned} &\frac{\partial}{\partial x'} = \frac{\partial}{\partial x} - \frac{\partial p}{\partial x}\frac{\partial}{\partial p}, \quad \frac{\partial}{\partial y'} = \frac{\partial}{\partial y} - \frac{\partial p}{\partial y}\frac{\partial}{\partial p}, \\ &\frac{\partial}{\partial p} = -\frac{1}{g\rho}\frac{\partial}{\partial z}, \quad \frac{\partial}{\partial t'} = \frac{\partial}{\partial t} - \frac{\partial p}{\partial t}\frac{\partial}{\partial p}, \end{aligned}\right\} \qquad (\ldots$$

The result is that

$$\frac{\partial p}{\partial x} = g\rho\frac{\partial z}{\partial x'}, \quad \frac{\partial p}{\partial y} = g\rho\frac{\partial z}{\partial y'}, \quad \frac{\partial p}{\partial t} = g\rho\frac{\partial z}{\partial t'}. \qquad (\ldots$$

In the new system the time derivative of any element has t following form:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} = \frac{\partial}{\partial t'} + u\frac{\partial}{\partial x'} + v\frac{\partial}{\partial y'} + \frac{dp}{dt}\frac{\partial}{\partial p} .$$

(1.1

This shows that in the system of coordinate x', y', p, t value $\tau = \frac{dp}{dt}$ plays the part of vertical velocity; this valu gives the position of the air unit in regard to the isobaric surface. The transition from w to τ is realized through th correlation

$$\tau = \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} - g\rho w$$

or

$$w = -\frac{1}{g\rho}\tau + \frac{\partial z}{\partial t'} + u\frac{\partial z}{\partial x'} + v\frac{\partial z}{\partial y'} , \qquad (1.$$

Equations (1.1) and (1.3) in the new coordinate system ar as follows:

$$\frac{\partial u}{\partial t'} + u\frac{\partial u}{\partial x'} + v\frac{\partial u}{\partial y'} + \tau\frac{\partial u}{\partial p} = -g\frac{\partial z}{\partial x'} + lv + g^2\frac{\partial}{\partial p}\rho\mu\frac{\partial u}{\partial p} ,$$

(1.

$$\frac{\partial v}{\partial t'} + u\frac{\partial v}{\partial x'} + v\frac{\partial v}{\partial y'} + \tau\frac{\partial u}{\partial p} = -g\frac{\partial z}{\partial y'} - lu - g^2\frac{\partial}{\partial p}\rho\mu\frac{\partial v}{\partial p} , \ (1$$

$$\frac{\partial z}{\partial p} = -\frac{1}{g\rho} . \qquad (1.1$$

In the system of the x', y', p,t' coordinate the equati[illegible] of continuity (1.4) can be greatly simplified. To this end we use the equation of statics (1.3) and correlations (1.12) and (1.9). Considering that

$$\frac{\partial \rho}{\partial s} = -\frac{1}{g}\frac{\partial}{\partial z}\left(\frac{\partial p}{\partial s}\right) \qquad (s = x,\ y,\ t),$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho w}{\partial y} = \frac{1}{g}\frac{\partial}{\partial z}\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y}\right) + \rho\frac{\partial \tau}{\partial p}$$

and

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \frac{1}{g}\left(u\frac{\partial}{\partial z}\frac{\partial p}{\partial x} + v\frac{\partial}{\partial z}\frac{\partial p}{\partial y}\right) =$$

$$= \rho\left(\frac{\partial u}{\partial x'} + \frac{\partial v}{\partial y'}\right) + \rho\left(\frac{\partial u}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial v}{\partial p}\frac{\partial p}{\partial y}\right) - \frac{1}{g}\left(u\frac{\partial}{\partial z}\frac{\partial p}{\partial x} + v\frac{\partial}{\partial z}\frac{\partial p}{\partial y}\right) =$$

$$= \rho\left(\frac{\partial u}{\partial x'} + \frac{\partial v}{\partial y'}\right) - \frac{1}{g}\frac{\partial}{\partial z}\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y}\right),$$

we obtain the following form (1.4)

$$\frac{\partial u}{\partial x'} + \frac{\partial v}{\partial y'} + \frac{\partial \tau}{\partial p} = 0 . \qquad (1$$

The equation of heat influx (1.5) is presented in the following manner if we take the new variables into account:

$$\frac{\partial T}{\partial t'} + u \frac{\partial T}{\partial x'} - v \frac{\partial T}{\partial y'} = \frac{\gamma_a - \gamma}{g\rho} \tau + \frac{\varepsilon}{c_p} , \qquad (1.17)$$

where

$$\gamma = - \frac{\partial T}{\partial z} .$$

Finally, we express the limit conditions (1.7) and (1.8) with the new variables as follows:

$$\tau = 0 \quad \text{with} \quad p = 0 \qquad (1.18)$$

and

$$\tau = g\rho_0 \frac{\partial z_0}{\partial t'} \quad \text{with} \quad p = p_0 . \qquad (1.19)$$

It can be easily recognized that the coordinate system selected by us, permits the use of the hydrodynamic equations of the atmosphere in the same way as the equations for the in compressible fluid.

Now after the problem of ascertainment of meteorological elements was formulated, we approach the problem of change to obtain equations suitable for physical conclusions.

For this purpose we differentiate equation (1.13) by y' and equation (1.14) by x'; we subtract the first equation from the second. We assume that ρu changes little in regard to the horizontal. Thus we obtain

$$\frac{\partial}{\partial t'}\left(\frac{\partial v}{\partial x'} - \frac{\partial u}{\partial y'}\right) + u\frac{\partial}{\partial x'}\left(\frac{\partial v}{\partial x'} - \frac{\partial u}{\partial y'}\right) + v\frac{\partial}{\partial y'}\left(\frac{\partial v}{\partial x'} - \frac{\partial u}{\partial y'}\right) +$$

$$+\left(\frac{\partial u}{\partial x'} + \frac{\partial v}{\partial y'}\right)\left(\frac{\partial v}{\partial x'} - \frac{\partial u}{\partial y'}\right) + \tau\frac{\partial}{\partial p}\left(\frac{\partial v}{\partial x'} - \frac{\partial u}{\partial y'}\right) + \frac{\partial v}{\partial p}\frac{\partial \tau}{\partial x'} -$$

$$-\frac{\partial u}{\partial p}\frac{\partial \tau}{\partial y'} = -l\left(\frac{\partial u}{\partial x'} + \frac{\partial v}{\partial y'}\right) - u\frac{\partial l}{\partial x'} - v\frac{\partial l}{\partial y'} + g^2\frac{\partial}{\partial p}\rho\mu\frac{\partial}{\partial p}$$

$$\left(\frac{\partial v}{\partial x'} - \frac{\partial u}{\partial y'}\right). \qquad (1.20)$$

Compared to other terms of the equation, components of type $\frac{\partial v}{\partial p}\frac{\partial \tau}{\partial x'}$ are generally small and can be disregarded.

We insert the following designations:

$$\frac{\partial v}{\partial x'} - \frac{\partial u}{\partial y'} = \Omega, \quad \frac{\partial u}{\partial x'} + \frac{\partial v}{\partial y'} = \mathrm{div}_p\,\vec{V}.$$

With the inclusion of new designations equation (1.20) take the following shape:

$$\frac{\partial(\Omega+l)}{\partial t'} + u\frac{\partial(\Omega+l)}{\partial x'} + v\frac{\partial(\Omega+l)}{\partial y'} - g^2\frac{\partial}{\partial p}\rho\mu\frac{\partial\Omega}{\partial p} =$$

$$= -(\Omega+l)\,\mathrm{div}_p\vec{V} - \tau\frac{\partial\Omega}{\partial p}. \qquad (1.21)$$

With the equation of continuity (1.16) we exclude $\mathrm{div}_p\vec{V}$ from (1.21). We obtain:

$$\frac{\partial(\Omega+l)}{\partial t'} + u\frac{\partial(\Omega+l)}{\partial x'} + v\frac{\partial(\Omega+l)}{\partial y'} - g^2\frac{\partial}{\partial p}\rho\mu\frac{\partial\Omega}{\partial p} =$$

$$= (\Omega+l)\frac{\partial\tau}{\partial p} - \tau\frac{\partial\Omega}{\partial p}. \qquad (1.22)$$

Inasmuch as value Ω is usually small compared to l, and inasmuch as the changes $\Omega+l$ which depend on the altitude, are small compared to the relative changes of τ according to altitude, we can simplify the equation to some extent as follows:

$$\frac{\partial(\Omega+l)}{\partial t'} + u\frac{\partial(\Omega+l)}{\partial x'} + v\frac{\partial(\Omega+l)}{\partial y'} - g^2\frac{\partial}{\partial p}\rho\mu\frac{\partial\Omega}{p} =$$

$$= l\frac{\partial\tau}{\partial p} \qquad (1.22$$

To simplify the analysis we check the dynamics of atmospheric motions first without the inclusion of frictional forces. The latter will be discussed at a later stage of the analysis.

For further simplification of equation (1.22) we assume that axis x' shows east and axis y' north. Then the following correlation is obtained:

$$\frac{\partial l}{\partial x'} = 0, \quad \frac{\partial l}{\partial y'} = \frac{2\omega \sin \Theta}{a_o} = \beta$$

a_o - radius of globe. In the following, parameter β will always be considered constant.

Taking into account all of the above conditions, equation (1.22) now takes the following form:

$$\frac{\partial \Omega}{\partial t'} + u\frac{\partial \Omega}{\partial x'} + v\frac{\partial \Omega}{\partial y'} + \beta v = l\frac{\partial \tau}{\partial p} . \qquad (1.23)$$

We put down the initial equations together with the limit conditions as follows:

$$\frac{du}{dt'} = - g\frac{\partial z}{\partial x'} + lv, \qquad (1.24$$

$$\frac{dv}{dt'} = - g\frac{\partial z}{\partial y'} - lu , \qquad (1.25$$

$$\frac{\partial \Omega}{\partial t'} + u\frac{\partial \Omega}{\partial x'} + v\frac{\partial \Omega}{\partial y'} + \beta v = l\frac{\partial \tau}{\partial p}, \qquad (1.2$$

$$\frac{\partial T}{\partial t'} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y'} = \frac{\gamma_a - \gamma}{g\rho}\tau + \frac{\varepsilon}{c_p}, \qquad (1.2$$

$$\frac{\partial z}{\partial p} = -\frac{1}{g\rho}, \qquad (1.$$

$$p = \rho RT, \qquad (1.$$

$$\tau = 0 \quad \text{with} \quad p = 0. \qquad (1.$$

$$\tau = g\rho_o\frac{\partial z_o}{\partial t'} \quad \text{with} \quad p = p_o. \qquad (1.$$

In the study of atmospheric motion for which the characte rizing dimensions in regard to the horizontal are in the ord of 1,000 kilometers, and the characteristic horizontal veloc ties is/10 sec/m, the analysis of equations (1.24) and (1.25) shows that these motions may be considered geostrophic motion

$$u = U + u^*,$$
$$v = V + v^*, \qquad (1.$$

where

$$U = -\frac{g}{l}\frac{\partial z}{\partial y'}, \quad V = \frac{g}{l}\frac{\partial z}{\partial x'} \tag{1.33}$$

- as component of geostrophic winds. u^*, v^* are small supplementing corrections.

The analysis of the expression for the vortex

$$\Omega = \frac{g}{l}\,\Delta z + \frac{\partial v^*}{\partial x'} - \frac{\partial u^*}{\partial y'} \quad ,$$

where

$$\Delta z = \frac{\partial^2 z}{\partial x'^2} + \frac{\partial^2 z}{\partial y'^2} ,$$

shows that value $\frac{g}{l}\Delta z$, i.e. the geostrophic approximation for the vorticity includes the main part of the vorticity.

We use the geostrophic approximation for the horizontal component of velocity u and v in equations (1.26) and (1.27); excluding ρ from (1.28) with (1.29) we obtain the following equations:

$$\frac{\partial \Delta z}{\partial t'} + \frac{g}{l}(z, \Delta z) + \beta\frac{\partial z}{\partial x'} = \frac{l^2}{g}\frac{\partial \tau}{\partial p} , \tag{1.34}$$

$$\frac{\partial T}{\partial t'} = \frac{g}{l}(T, z) + \frac{\varepsilon}{c_p} + \frac{R(\gamma_a - \gamma)}{g}\frac{T}{p}\tau , \tag{1.35}$$

$$\frac{\partial z}{\partial p} = - \frac{R}{g} \frac{T}{p} \ , \tag{1.36}$$

$$(A,B) = \frac{\partial A}{\partial x'} \frac{\partial B}{\partial y'} - \frac{\partial A}{\partial y'} \frac{\partial B}{\partial x'} \ .$$

Together with limit conditions (1.30) and (1.31) and elementary data this will suffice to determine future pressure fields temperature and vertical currents.

Today the ascertainment of the general solution of the non-linear system for the differential equations (1.34) to (1.36) meets difficulties which cannot be overcome. We have only taken up the problem to ascertain the first derivatives $\frac{\partial z}{\partial t}$, $\frac{\partial T}{\partial t}$ and function τ according to the given distribution of the pressure fields and the temperature.

2. Equation for Pressure Change

We record the equation system (1.34) to (1.36) in the following manner:

$$\frac{\partial \Delta z}{\partial t} + \frac{g}{l} (z, \Delta z) + \beta \frac{\partial z}{\partial x} = \frac{l^2}{Pg} \frac{\partial \tau}{\partial \zeta} \quad {}^{1} \ ; \tag{2.1}$$

[1] Index (') in the coordinates x, y, t is left out.

$$\frac{\partial T}{\partial t} - \frac{g}{l}(T,\ z) - \frac{\varepsilon}{c_p} = \frac{m^2 l^2}{PR}\,\frac{\tau}{\zeta} \quad ; \qquad (2.2)$$

$$T = -\frac{g}{R}\,\zeta\,\frac{\partial z}{\partial \zeta} \quad , \qquad (2.$$

where $\zeta = \frac{p}{P}$ and P the mean pressure on the surface of the earth which was assumed with 1,000 millibar.

$$m^2 = \frac{R^2\,T(\gamma_a - \gamma)}{g\,l^2} \quad .$$

We assume that parameter m^2 changes *with respect to height* to a very small extent compared to the relative changes with the altitude of vertical velocity τ.

From equation (2.2) we obtain the derivative of temperat $\frac{\partial T}{\partial t}$ through the equation of statics. Thus we obtain:

$$-\frac{g}{l}\,\zeta\,\frac{\partial}{\partial \zeta}\left(\frac{\partial z}{\partial t}\right) - \frac{g}{l}(T,\ z) - \frac{\varepsilon}{c_p} = \frac{m^2 l^2}{PR}\,\frac{\tau}{\zeta} \quad . \qquad (2.4$$

Both parts of the equation are multiplied by ζ. Then we differentiate by ζ. Thus we obtain:

$$-\frac{g}{R}\,\frac{\partial}{\partial \zeta}\,\zeta^2\,\frac{\partial}{\partial \zeta}\left(\frac{\partial z}{\partial t}\right) - \frac{\partial}{\partial \zeta}\,\zeta\left[\frac{g}{l}(T,\ z) + \frac{\varepsilon}{c_p}\right] = \frac{m^2 l^2}{PR}\,\frac{\partial \tau}{\partial \zeta} . \quad ($$

Excluding derivative $\frac{\partial \tau}{\partial \zeta}$ from equations (2.1) and (2.5) we obtain

$$\left(\frac{\partial}{\partial \zeta} \zeta^2 \frac{\partial}{\partial \zeta} + m^2 \Delta\right) \frac{\partial z}{\partial t} = f_1 (x, y, \zeta), \qquad (2.6$$

where

$$f_1 (x, y, \zeta) = - m^2 \left[\frac{g}{l} (z, \Delta z) + \beta \frac{\partial z}{\partial x}\right] - \frac{R}{g} \frac{\partial}{\partial \zeta} \zeta \left[\frac{g}{l} (T, z) + \frac{\varepsilon}{c_p}\right]. \qquad (2.7$$

In the cylindrical coordinate system (r, ϕ, ζ) where $r = \frac{\sqrt{x^2 + y^2}}{m}$, ϕ - ~~zenithic~~ polar angle and $\zeta = \frac{p}{P}$ the reduced altitude, equation (2.6) is changed in the following manner:

$$\left(\frac{\partial}{\partial \zeta} \zeta^2 \frac{\partial}{\partial \zeta} + \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2}\right) \frac{\partial z}{\partial t} = f_1(r, \phi, \zeta). \qquad (2.$$

To formulate the problem completely, equation (2.8) shou[l]
be supplemented by limit conditions.

As the first limit condition we assume the relation direc[t]ly resulting from the heat influx equation (2.2) on the surf[ace] level of 1,000 millibar $(\zeta = 1)$ on the basis of condition (1.31) as follows:

$$\frac{\partial T_o}{\partial t} = \frac{g}{l}(T_o, z_o) + \frac{\varepsilon_o}{c_p} + (\gamma_a - \gamma) \frac{\partial z_o}{\partial t}. \qquad (2.$$

Considering that $T_o = -\left(\frac{g}{R}\zeta\frac{\partial z}{\partial \zeta}\right)_{\zeta = 1}$, we put (2.9) into the following shape:

$$\left(\zeta\frac{\partial}{\partial \zeta} + \alpha\right)\frac{\partial z}{\partial t}\bigg|_{\zeta = 1} = - A(r, \phi, 1), \qquad (2.$$

where

$$A(r, \phi, 1) = \frac{R}{g}\left[\frac{g}{l}(T_o, z_o) + \frac{\varepsilon_o}{c_p}\right], \qquad (2.$$

$$\alpha = \frac{R(\gamma_a - \gamma)}{g} \approx 0,1. \qquad (2.$$

The second limit condition is obtained from the heat influx equation for the upper limit of the atmosphere in rega to the volume unit of the air:

$$\lim_{\zeta \to 0}\left(c_p\rho\frac{\partial T}{\partial t}\right) = \lim_{\zeta \to 0}\left[c_p\rho\frac{g}{l}(T, z) + \varepsilon' + \frac{c_p(\gamma_a - \gamma)}{g}\tau\right], \quad (2.$$

ε' - influx of heat at volume unit.

On the basis of (1.30) we have

$$\lim_{\zeta \to 0}\left[\frac{c_p(\gamma_a - \gamma)}{g}\tau\right] = 0.$$

Moreover, the quantity of heat ε' flowing at the volum unit, is very small in the upper atmospheric layer. Therefo

$$\lim_{\zeta \to 0} \varepsilon' = 0.$$

Finally, $\lim_{\zeta \to 0} \rho u = \lim_{\zeta \to 0} \rho v = 0$ and the derivatives of temperature $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial y}$ are BOUNDED ON the isobaric surfaces in regard to the large-scale motions; as a result

$$\lim_{\zeta \to 0} \left[c_p \rho \frac{g}{l} (T, z) \right] = 0.$$

In this way we obtain the relation

$$\lim_{\zeta \to 0} \left(c_p \rho \frac{\partial T}{\partial t} \right) = 0. \tag{2.14}$$

Condition (2.14) can be put down in the following satisfactory manner if we take (2.3) into account:

$$\zeta^2 \frac{\partial}{\partial \zeta} \left(\frac{\partial z}{\partial t} \right)_{\zeta \to 0} = 0. \tag{2.15}$$

In this manner the work done for the ascertainment of the first derivatives according to time led from the altitude of isobaric surfaces to the integration of the inhomogeneous, differential equation of the second order (2.8) under limit conditions (2.10) and (2.15).

Now we proceed to the problem (2.8), (2.10),(2.15). We assume that functions $f_1(r, \varphi, \zeta)$ and $A(r, \varphi, 1)$ can be presented in the following form:

$$\left.\begin{aligned} f_1(r, \varphi, \zeta) &= \mathrm{Re} \sum_{n=-\infty}^{\infty} e^{in\varphi} \int_0^{\infty} F_n(\rho, \zeta) J_n(r\rho)\rho d\rho, \\ A(r, \varphi, 1) &= \mathrm{Re} \sum_{n=-\infty}^{\infty} e^{in\varphi} \int_0^{\infty} G_n(\rho) J_n(r\rho)\, \rho\, d\rho, \end{aligned}\right\} \quad (2.16$$

where

$$\left.\begin{aligned} F_n &= \frac{1}{2\pi} \int_0^{2\pi} e^{-in\varphi} d\varphi \int_0^{\infty} f_1(r', \varphi, \zeta) J_n(\rho r') r' dr', \\ G_n &= \frac{1}{2\pi} \int_0^{2\pi} e^{-in\varphi} d\varphi \int_0^{\infty} A(r', \varphi, 1) J_n(\rho r') r' dr'. \end{aligned}\right\} \quad (2.1$$

Here $J_n(x)$ is Bessel's function of the n-order, and the symbol Re means that only the "Real" (essential) parts of the corresponding expressions are examined.

We look for the solution of the problem in the following manner:

$$\frac{\partial z}{\partial t} = \mathrm{Re} \sum_{n=-\infty}^{\infty} e^{in\varphi} \int_0^{\infty} S_n(\rho, \zeta) J_n(r\rho) \rho d\rho . \quad (2.$$

We subject (2.18) and (2.16) under limit conditions (2.10) and (2.15) in equation (2.8). We combine the terms containing $e^{in\phi} J_n(r\rho)$ products with the same n, and adapt them to zero. We use correlation

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} J_n(\rho r) - \frac{n^2}{r^2} J_n(\rho r) = -\rho^2 J_n(\rho r),$$

As a result we obtain the following equation and the limit conditions for a new unknown function $S_n(\rho, \zeta)$:as follows:

$$\frac{\partial}{\partial \zeta} \zeta^2 \frac{\partial S_n}{\partial \zeta} - \rho^2 S_n = F_n(\rho, \zeta), \tag{2.19}$$

$$\left(\zeta \frac{\partial}{\partial \zeta} + \alpha \right) S_n \Big|_{\zeta=1} = - G_n(\rho), \tag{2.20}$$

$$\zeta^2 \frac{\partial S_n}{\partial \zeta} \Big|_{\zeta=0} = 0. \tag{2.21}$$

The solution of equation (2.19) is composed of the general solution of the homogeneous equation

$$\zeta^2 \frac{\partial^2 S_n}{\partial \zeta^2} + 2\zeta \frac{\partial S_n}{\partial \zeta} - \rho^2 S_n = 0 \tag{2.22}$$

and of the specific solution of inhomogenous equation (2.19). The general solution of homogenous equation (2.22) has the following form:

$$S_n^o(\rho,\zeta) = C_{n1}\zeta^{\nu_1} + C_{n2}\zeta^{\nu_2}, \qquad (2.2$$

where

$$\nu_1 = -\frac{1}{2} + \sqrt{\frac{1}{4} + \rho^2}, \quad \nu_2 = -\frac{1}{2} - \sqrt{\frac{1}{4} - \rho^2}, \qquad (2.$$

C_{n1} and C_{n2} - constant quality depending upon ρ.

The particular solution $S_n^*(\rho,\zeta)$ of the inhomogenous tion (2.19) will be found through the method of variation of constants.

$$S_n^*(\rho,\zeta) = D_{n1}(\rho,\zeta)\zeta^{\nu_1} + D_{n2}(\rho,\zeta)\zeta^{\nu_2}. \quad (2.$$

As it is known functions D_{n1} and D_{n2} are determined from following equation system:

$$\left.\begin{aligned} D'_{n1}\zeta^{\nu_1} + D'_{n2}\zeta^{\nu_2} &= 0, \\ \nu_1 D'_{n1}\zeta^{\nu_1-1} + \nu_2 D'_{n2}\zeta^{\nu_2-1} &= \frac{F_n}{\zeta^2}, \end{aligned}\right\} \qquad (2.$$

$$D'_{ni} = \frac{D_{ni}}{\partial \zeta} .$$

As a result of the solution of system (2.26) we obtain:

$$D'_{n1} = \frac{1}{\nu_1 - \nu_2} F_n \zeta^{\nu_2} , \quad D'_{n2} = - \frac{1}{\nu_1 - \nu_2} F_n \zeta^{\nu_1} . \qquad (2.27)$$

We integrate (2.27) at the limits from zero to ζ. The ar bitrarily chosen integration constants are excluded and we ob tain:

$$D_{n1}(\rho, \zeta) = \frac{1}{\nu_1 - \nu_2} \int_0^{\zeta} F_n(\rho, \eta) \eta^{\nu_2} \, d\eta ,$$

$$D_{n2}(\rho, \zeta) = - \frac{1}{\nu_1 - \nu_2} \int_0^{\zeta} F_n(\rho, \eta) \eta^{\nu_1} \, d\eta .$$

Consequently, the particular solution of the inhomogenous equation has the following form:

$$S_n^{*} = \frac{1}{\nu_1 - \nu_2} \int_0^{\zeta} F_n(\rho, \eta) \left[\zeta^{\nu_1} \eta^{\nu_2} - \zeta^{\nu_2} \eta^{\nu_1} \right] d\eta . \quad (2.2$$

In this manner we obtain the desired general solution of equa tion (2.19)

$$S_n(\rho,\zeta) = C_{n1}\zeta^{\nu_1} + C_{n2}\zeta^{\nu_2} + \frac{1}{\nu_1-\nu_2}\int_0^{\zeta} F_n(\rho,\eta)\left[\zeta^{\nu_1}\eta^{\nu_2} - \zeta^{\nu_2}\eta^{\nu_1}\right] d\eta . \qquad (2.2$$

C_{n1} and C_{n2} are obtained from the limit conditions (2.20) and (2.21). The result of condition (2.21) is that

$$C_{n2} = 0. \qquad (2.3$$

Value C_{n1} can be determined from condition (2.20)

$$\nu_1 C_{n1} + \frac{1}{\nu_1-\nu_2}\int_0^1 F_n(\rho,\eta)\left[\nu_1\eta^{\nu_2} - \nu_2\eta^{\nu_1}\right] d\eta + \alpha C_{n1} + \frac{\alpha}{\nu_1-\nu_2}\int_0^1 F_n(\rho,\eta)\left[\eta^{\nu_2} - \eta^{\nu_1}\right] d\eta = - G_n(\rho),$$

resulting in

$$C_{n1} = \frac{1}{\nu_1-\nu_2}\int_0^1 F_n(\rho,\eta)\left[\eta^{\nu_1} - \eta^{\nu_2}\right] d\eta - \frac{1}{\alpha+\nu_1}\int_0^1 F_n(\rho,\eta)\,\eta^{\nu_1} d\eta - \frac{G_n(\rho)}{\alpha+\nu_1} . \qquad (2.3$$

On the basis of (2.30) and (2.31) in (2.29) we obtain

$$S_n(\rho,\zeta) = -\frac{1}{\nu_1-\nu_2}\int_0^1 F_n(\rho,\eta)\Big[\delta_1\zeta^{\nu_2}\eta^{\nu_1} + \delta_2\zeta^{\nu_1}\eta^{\nu_2} -$$

$$-(\zeta\eta)^{\nu_1}\Big]\,d\eta - \frac{1}{\alpha+\nu_1}\int_0^1 F_n(\rho,\eta)\zeta^{\nu_1}\eta^{\nu_1}d\eta - \frac{G_n(\rho)}{\alpha+\nu_1}\zeta^{\nu_1}, \tag{2.}$$

where

$$\delta_1 = \begin{cases} 1 \text{ with } \eta \le \zeta, \\ 0 \text{ with } \eta > \zeta, \end{cases} \qquad 2 = \begin{cases} 0 \text{ with } \eta \le \zeta, \\ 1 \text{ with } \eta > \zeta. \end{cases}$$

Now we take (2.32) as a basis for (2.18). We obtain:

$$\frac{\partial z}{\partial t} = -\operatorname{Re}\sum_{n=-\infty}^{\infty} e^{in\phi}\int_0^\infty \Bigg\{\frac{1}{2\mu}\int_0^1 F_n(\rho,\eta)\frac{1}{\sqrt{\zeta\eta}}\Big[\delta_1\Big(\frac{\eta}{\zeta}\Big)^\mu +$$

$$+\delta_2\Big(\frac{\zeta}{\eta}\Big)^\mu - (\zeta\eta)^\mu\Big]\,d\eta + \frac{1}{\alpha-\frac{1}{2}+\mu}\int_0^1 F_n(\rho,\eta)\frac{1}{\sqrt{\zeta\eta}}(\zeta\eta)^\mu\, d$$

$$-\frac{G_n(\rho)}{\alpha-\frac{1}{2}+\mu}\frac{\zeta^\mu}{\sqrt{\zeta}}\Bigg\} J_n(r\rho)\rho\,d\rho, \tag{2.33}$$

where $\mu = \sqrt{\frac{1}{4}+\rho^2}$.

We also take the values of $F_n(\rho,\eta)$ and $G_n(\rho)$ from (2.1 as a basis for (2.33) and change the sequence of integratio

The result is as follows:

$$\frac{\partial z}{\partial t} = -\frac{1}{2\pi} \operatorname{Re} \sum_{n=-\infty}^{\infty} e^{in\phi} \int_0^{\infty} \int_0^{2\pi} e^{-in\psi} \left\{ \int_0^1 f_1(r', \psi, \eta)\, M_1^{(n)} \right.$$

$$\left. (r, \zeta, r', \eta) d\eta + A(r', \psi, 1) M^{*(n)}(r, \zeta, r') \right\} r' d\psi dr', \quad (\text{:}$$

where

$$M_1^{(n)}(r, \zeta, r', \eta) = \frac{1}{2\sqrt{\zeta\eta}} \int_0^{\infty} \left[\delta_1 \left(\frac{\eta}{\zeta}\right)^{\mu} + \delta_2 \left(\frac{\zeta}{\eta}\right)^{\mu} - (\zeta\eta)^{\mu} \right]$$

$$\frac{J_n(r\rho) J_n(r'\rho)}{\mu} \rho d\rho + \frac{1}{\sqrt{\zeta\eta}} \int_0^{\infty} (\zeta\eta)^{\mu} \frac{J_n(r\rho) J_n(r'\rho)}{\alpha - \frac{1}{2} + \mu} \rho d\rho,$$

$$M^{*(n)}(r, \zeta, r') = \frac{1}{\sqrt{\zeta}} \int_0^{\infty} \zeta^{\mu} \frac{J_n(r\rho) J_n(r'\rho)}{\alpha - \frac{1}{2} + \mu} \rho d\rho . \quad ($$

If we take the vertical line crossing the zero of the co-ordinate system $(r = 0)$[1] as a testing point, we get $n \neq 0$ all values if $J_n(0) = 0$ with $n \neq 0$ and $J_o(0) = 1$ are taken into account.

$$M_1^{(n)} = 0 \quad \text{and} \quad M^{*(n)} = 0$$

[1] We can always combine the vertical axis of the coordinate the studied vertical.

Only functions $M_1^{(o)}$ and $M^{*(o)}$ differ from zero.

In this manner, the sumtotal (2.34) is transformed to a composed solution, with r = 0.

$$\frac{\partial z}{\partial t}\bigg|_{r=0} = -\int_0^\infty\int_0^1 F_1(r',\eta)M_1(\zeta, r',\eta)r'd\eta dr' -$$

$$-\int_0^\infty A(r', 1)M^*(\zeta, r')r'dr \, . \qquad (2.37$$

$$\left.\begin{aligned} F_1(r',\eta) &= \frac{1}{2\pi}\int_0^{2\pi} f_1(r',\psi,\eta)d\psi \, , \\ A(r', 1) &= \frac{1}{2\pi}\int_0^{2\pi} A(r',\psi, 1)d\psi \, , \end{aligned}\right\} \qquad (2.38$$

$$M^*(x, r) = \frac{1}{\sqrt{x}}\int_0^\infty x^\mu \frac{J_o(r'\rho)}{\alpha - \frac{1}{2} + \mu}\rho d\rho \, , \qquad (2.39$$

$$\left.\begin{aligned} M_1(\zeta, r',\eta) &= M^*(\zeta\eta, r') + M_3(\zeta, r',\eta), \\ M_3(\zeta, r',\eta) &= \frac{1}{2\sqrt{\zeta\eta}}\int_0^\infty \left[\delta_1\left(\frac{\eta}{\zeta}\right)^\mu + \delta_2\left(\frac{\zeta}{\eta}\right)^\mu - (\zeta\eta)^\mu\right] \\ &\qquad \frac{J_o(r'\rho)}{\mu}\rho d\rho \, . \end{aligned}\right\} \qquad (2.40$$

The result of (2.37) is that the pressure change at the points which are situated on any vertical line, is determined by the mean value of the exact functions of the pressure and temperature fields, in the circumferences on the isobaric surfaces, the center being on the examined vertical.

This result can be obtained through a simpler mathematical process. We integrate equation (2.8) with φ, from zero to 2π. We find the solution for the derivatives $\frac{\partial z}{\partial t}$, obtained with the circumference of radius r; thus we assume that in the obtained solution $r = 0$.

In the following this method will be applied for the obtainment of the solution for $\frac{\partial T}{\partial t}$ and τ.

Now we concentrate on the fact that according to (2.7) the "thermal coefficients" $\frac{g}{l}(T, z) + \frac{\varepsilon}{c_p}$ enter the solution (2.37) in a very complicated manner as

$$f_1(r', \psi, \eta) = - B(r', \psi, \eta) - \frac{\partial}{\partial \eta} \eta A(r', \psi, \eta),$$

where

$$\left.\begin{aligned} B(r', \psi, \eta) &= m^2 \left[\frac{g}{l}(z, \Delta z) + \beta \frac{\partial z}{\partial x}\right], \\ A(r', \psi, \eta) &= \frac{R}{g}\left[\frac{g}{l}(T, z) + \frac{\varepsilon}{c_p}\right]. \end{aligned}\right\} \qquad (2.$$

To simplify solution (2.37) we try to find the function of influence referring directly to $A(r', \eta)$. For this purpose we use expression $-\frac{\partial}{\partial \eta} \eta A(r', \eta)$ instead of $F_1(r', \eta)$ in (2.37). We integrate in stages and obtain:

$$\int_0^1 \frac{\partial}{\partial \eta} \eta A(r', \eta) M_1(\zeta, r', \eta) d\eta - A(r', 1) M^*(\zeta, r') =$$

$$= \eta A(r', \eta) M_1(\zeta, r', \eta) \Big|_0^1 - \int_0^1 A(r', \eta) \eta \frac{\partial}{\partial \eta} M_1(\zeta, r', \eta) d\eta$$

$$- A(r', 1) M^*(\zeta, r') = -\int_0^1 A(r', \eta) \eta \frac{\partial}{\partial \eta} M_1(\zeta, r', \eta) d\eta =$$

$$= \int_0^1 A(r', \eta) M_2(\zeta, r', \eta) d\eta , \qquad (2.$$

where

$$\left.\begin{aligned} M_2(\zeta, r', \eta) &= -\eta \frac{\partial}{\partial \eta} M_1 = N(\zeta, r', \eta) + \\ &- \frac{1}{2} M_3(\zeta, r', \eta) + \alpha M^*(\zeta\eta, r'), \\ N(\zeta, r', \eta) &= \frac{1}{2\sqrt{\zeta\eta}} \int_0^\infty \left[\delta_2 \left(\frac{\zeta}{\eta}\right)^\mu - \delta_1 \left(\frac{\eta}{\zeta}\right)^\mu - (\zeta\eta)^\mu \right] \\ &\qquad J_0(r'\rho)\, \rho d\rho . \end{aligned}\right\} \qquad (2.$$

Considering (2.41) and (2.42) we, finally, present the solution in the following form:

$$\frac{\partial z}{\partial t} = \frac{1}{2\pi}\int_0^1\int_0^\infty\int_0^{2\pi} m^2\left[\frac{g}{l}(z, \Delta z) + \beta\frac{\partial z}{\partial x}\right]M_1(\zeta, r, \eta)\, r\, d\varphi\, dr\, d\eta +$$

$$+ \frac{1}{2\pi}\int_0^1\int_0^\infty\int_0^{2\pi} \frac{R}{g}\left[\frac{g}{l}(T, z) + \frac{\varepsilon}{c_p}\right] M_2(\zeta, r, \eta)\, r\, d\varphi\, dr\, d\eta . \quad (2.4$$

The influence function $M_1(\zeta, r, \eta)$ and $M_2(\zeta, r, \eta)$ is characterized by the dependence area of the solution of the meteorological element fields in the surrounding space.

To calculate functions of M^*, M_1 and M_2 it is appropria to make certain changes in (2.39), (2.41) and (2.43). The values $(\zeta\eta)^\mu$, $\left(\frac{\eta}{\zeta}\right)^\mu$ and $\left(\frac{\zeta}{\eta}\right)^\mu$ are presented as instructive functions of type e^{-x}:

$$(\zeta\eta)^\mu = e^{-\mu \ln\frac{1}{\zeta\eta}}, \quad \left(\frac{\eta}{\zeta}\right)^\mu = e^{-\mu\ln\frac{\zeta}{\eta}}, \quad \left(\frac{\zeta}{\eta}\right)^\mu = e^{-\mu\ln\frac{\eta}{\zeta}}$$

With the inequalities

$$\zeta\eta \leq 1, \quad \frac{\eta}{\zeta} \leq 1, \quad \frac{\zeta}{\eta} \leq 1$$

the functions $\ln\frac{1}{\zeta\eta}$, $\ln\frac{\zeta}{\eta}$ and $\ln\frac{\eta}{\zeta}$ are always pos tive.

As a result function $M_3(\zeta, r, \eta)$ is converted to the algebraic sumtotal of three integrals of type (4):

$$\sigma(x, r) = \int_0^\infty e^{-\mu \ln \frac{1}{x}} \frac{J_0(r\rho)}{\mu} \rho d\rho = \frac{e^{-\frac{1}{2}\sqrt{\ln^2 \frac{1}{x} + r^2}}}{\sqrt{\ln^2 \frac{1}{x} + r^2}}, \tag{2.45}$$

where $\mu = \sqrt{\frac{1}{4} + \rho^2}$, and x takes value $\zeta\eta$, $\frac{\eta}{\zeta}$, $\frac{\zeta}{\eta}$.

If we use the symbols of (2.45), function $M_3(\zeta, r, \eta)$ is presented in the following manner:

$$M_3(\zeta, r, \eta) = \frac{1}{2\sqrt{\zeta\eta}} \left[\delta_1 \sigma\left(\frac{\eta}{\zeta}, r\right) + \delta_2 \sigma\left(\frac{\zeta}{\eta}, r\right) - \sigma(\zeta\eta, r)\right]. \tag{2.46}$$

To calculate $M^*(x, r)$ we use the transformation

$$\int_0^\infty x^{\mu - \frac{1}{2}} \frac{J_0(r\rho)}{\alpha - \frac{1}{2} + \mu} \rho d\rho = x^{-\alpha} \int_0^x \lambda^{\alpha - \frac{1}{2}} \frac{\partial}{\partial\lambda} \left\{ \int_0^\infty \lambda^{\mu} \frac{J_0(r\rho)}{\mu} \rho d\rho \right\} d\lambda.$$

Considering (2.45) we obtain:

$$M^*(x, r) = x^{-\alpha} \int_0^x \lambda^{\alpha - \frac{1}{2}} \frac{\partial\sigma(\lambda, r)}{\partial\lambda} d\lambda =$$

$$= \frac{\sigma(x, r)}{\sqrt{x}} + \left(\frac{1}{2} - \alpha\right) x^{-\alpha} \int_0^x \lambda^{\alpha - \frac{3}{2}} \sigma(\lambda, r) d\lambda. \tag{2.47}$$

We introduce

$$\frac{\sigma(x, r)}{\sqrt{x}} = U(x, r). \tag{2.4}$$

As a result we can present function $M_1(\zeta, r, \eta)$ through the following expression:

$$M_1(\zeta, r, \eta) = \frac{1}{2}\left[\frac{\delta_1}{\zeta} U\left(\frac{\eta}{\zeta}, r\right) + \frac{\delta_2}{\eta} U\left(\frac{\zeta}{\eta}, r\right) + U(\zeta\eta, r)\right]$$

$$+ \left(\frac{1}{2} - \alpha\right)(\zeta\eta)^{-\alpha}\int_0^{\zeta\eta} x^{\alpha-1} U(x, r)\,dx, \tag{2.49}$$

ζ - the level for which $\frac{\partial z}{\partial t}$ is determined, r, η - the variable quantities of integration.

To calculate function $N(\zeta, r, \eta)$ we use the transformati

$$\int_0^\infty x^{\mu-\frac{1}{2}} J_0(r\rho)\,\rho\,d\rho = \sqrt{x}\,\frac{\partial}{\partial x}\int_0^\infty x^{\mu}\,\frac{J_0(r\rho)}{\mu}\,\rho\,d\rho \tag{2.50}$$

and then also correlations (2.45) and (2.48). Then $N(\zeta, r, \eta)$ becomes the sumtotal of three integrals of type

$$I(x, r) = \int_0^\infty x^{\mu-\frac{1}{2}} J_0(r\rho)\,\rho\,d\rho = \sqrt{x}\,\frac{\partial\sigma(x, r)}{\partial x} =$$

$$= \frac{1}{2}\,\frac{\ln\frac{1}{x}\left(2 + \sqrt{\ln^2\frac{1}{x} + r^2}\right)}{\ln^2\frac{1}{x} + r^2}\,U(x, r). \tag{2.5}$$

Taking note of (2.46), (2.48), and (2.51) function M_2 can now be described in the following manner:

$$M_2(\zeta, r, \eta) = \frac{1}{2}\left\{\frac{\delta_2}{\eta} I\left(\frac{\zeta}{\eta}, r\right) - \frac{\delta_1}{\zeta} I\left(\frac{\eta}{\zeta}, r\right) - I(\zeta\eta, r)\right\} +$$

$$+ \frac{1}{4}\left\{\frac{\delta_1}{\zeta} U\left(\frac{\eta}{\zeta}, r\right) + \frac{\delta_2}{\eta} U\left(\frac{\zeta}{\eta}, r\right) - U(\zeta\eta, r)\right\} + \alpha M^*(\zeta\eta, r). \quad (2.52$$

We note particularly that with $\zeta = 1$, the influence function M_1 and M_2 has a usual form as follows:

$$M_1(1, r, \eta) = M^*(\eta, r), \quad (2.53$$

$$M_2(1, r, \eta) = -I(\eta, r) + \alpha M^*(\eta, r). \quad (2.54)$$

Functions $U(x, r)$, $I(x, r)$, $M^*(x, r)$ and the fields of functions $M_1(\zeta, r, \eta)$ and $M_2(\zeta, r, \eta)$ for different cutting through the vertical plane, are graphically shown in Fig. 1 - 12.

We will analyse the reaction of these functions with different values of the independent variables x, r, η.

Function $U(x, r)$ (Fig. 1) is $U(1, r) = \frac{1}{r} e^{-\frac{r}{2}}$ with $x = 1$, and $U(0, r) = 0$ with $x = 0$; with a small x the functi[on] tends toward zero such as $\frac{1}{\ln \frac{1}{x}}$. With $r = 0$ $U(x, 0) = \frac{1}{\ln}$

with growing r and a fixed value x function $U(x, r)$ decrease in a regular manner; with $r \rightarrow \infty$ it tends toward zero, as $\frac{1}{r} e^{-\frac{r}{2}}$. The function shows its characteristic trait at po[illegible] $(x = 1, r = 0)$.

With $x = 1$ and $r \neq 0$ function $I(x, r)$ (Fig. 2) equals zero; with $x = 0$ $I(0, r) = 0$; with a small x the function tends toward zero, as $\frac{1}{2 \ln \frac{1}{x}}$. With $r = 0$ $I(x, 0) = \frac{2 + \ln \frac{1}{x}}{2 \ln^2 \frac{1}{x}}$. With growing r and a fixed value x function $I(x, r)$ decreases in a regular manner; with $r \rightarrow \infty$ it tends toward zero, as $\frac{1}{2r^2} e^{-\frac{r}{2}}$. The function is characteristic at point $(x = 1, r = 0)$.

The characteristics of function $M^*(x, r)$ (Fig. 3) are similar to function $U(x, r)$. Values $M^*(x, r)$ are 1.5 to two times higher than values $U(x, r)$.

With $x = 0$ $M^*(0, r) = 0$; with $r \rightarrow \infty$ $M^*(x, r)$ appear[illegible] as $\frac{1}{r} e^{-\frac{r}{2}}$ (Fig. 4). The tendency of the function toward zero is governed by the rule $\frac{x}{\ln \frac{1}{x}}$.

Function $M_1(1, r, \eta)$ Fig. 5) describes the integral act[illegible] of "the dynamic factors" $\frac{g}{l}(z, \Delta z) + \beta \frac{\partial z}{\partial x}$ in regard to the pressure change at the point on the earth's surface. The act[illegible] distance $R = \sqrt{m^2} \approx 750$ km corresponds to the relative length

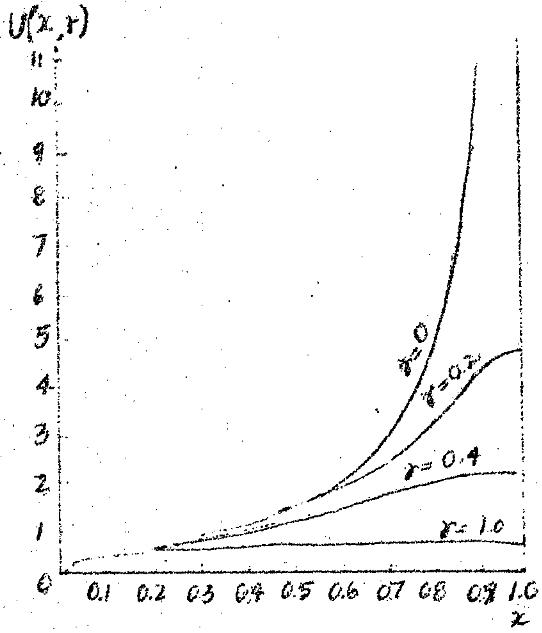

Fig. 1
Graphic Presentation of Function U(x , r) with Different r Values

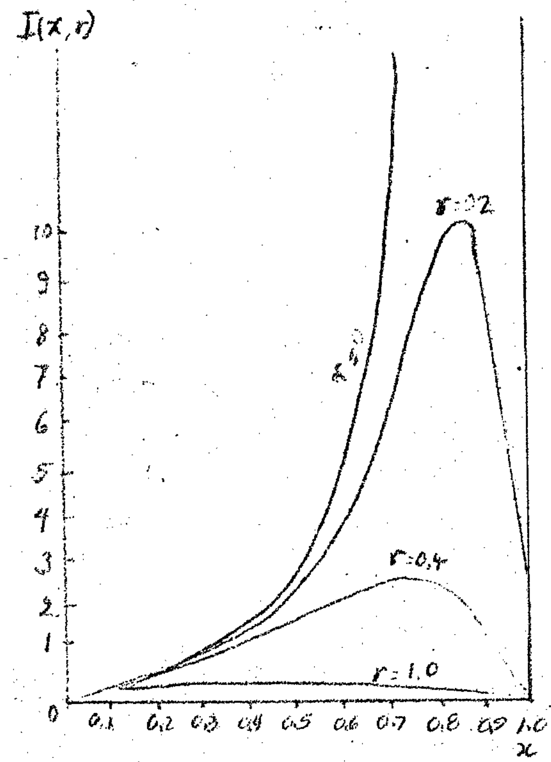

Fig. 2
Graphic Presentation of Function I(x , r) with Different r Values

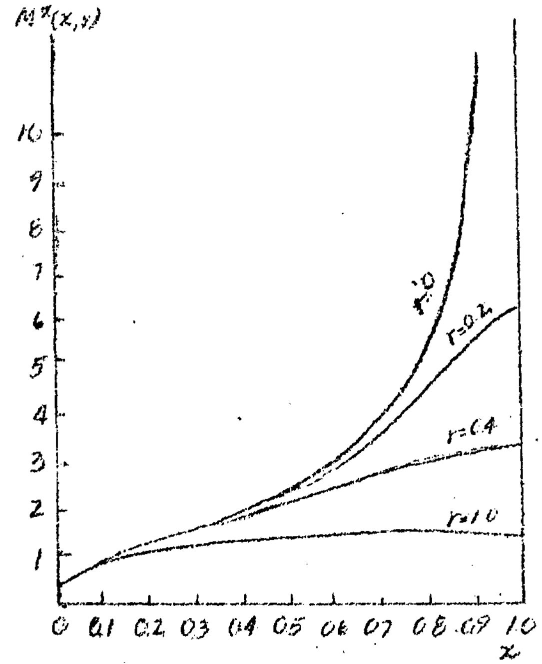

Fig. 3. Graphic Presentation
of Function M^* (x , r) with Different
r Values

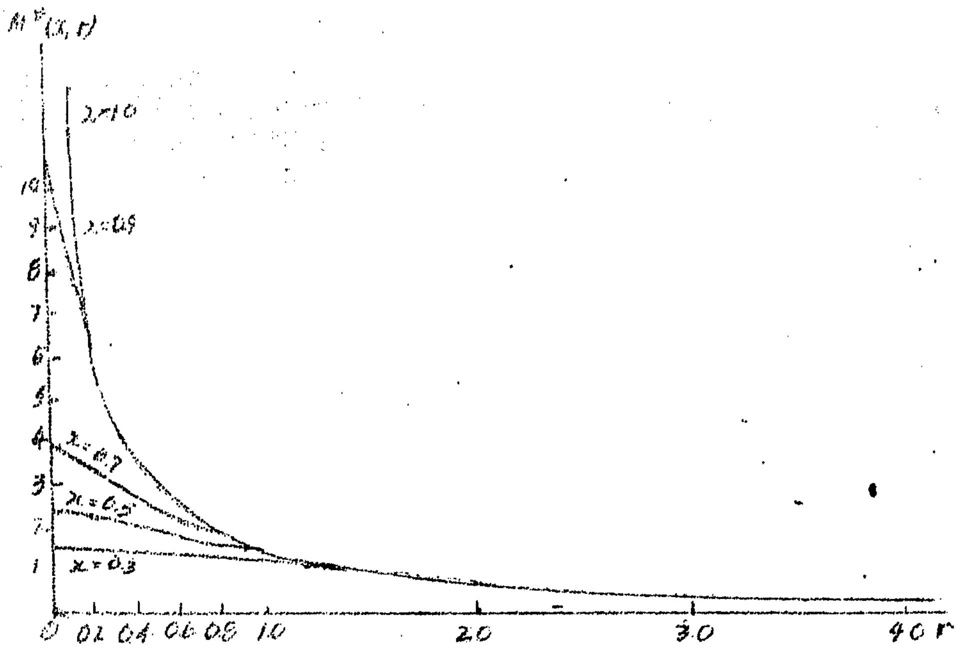

Fig. 4. Graphic Presentation
of Function M^* (x , r) with Varying .

Attention must be given to the vast expansion of the action area of the dynamic factors in horizontal direction. For better illustration and analyzis of vanishing of action on the part of the dynamic factors with increasing r values, we will show the influence of $rM_1 (1, r, \eta)$, in a graphical manner, describing the importance of average values of dynamic factors $\frac{g}{l} (z, \Delta z) + \beta \frac{\partial z}{\partial x}$ relative to circumferences of radius r (Fig. 6). We see that function rM_1 vanishes very slowly with increasing r. With $r \to \infty$ $rM_1(1, r, \eta)$ decreas as $e^{-\frac{r}{2}}$.

In actual practice, the influence of dynamic factors on the change of surface pressure will always be limited by radius R of the 2,000 km order, because the average values o

$$\frac{g}{l} (z, \Delta z) + \beta \frac{\partial z}{\partial x}$$ relative to the circumferences 2,000 - 3,000 km are actually small.

The limited effect on the change of surface pressure and dynamic factors in the upper layers, can be seen in a cleare manner in Fig. 6. With an equal distribution of the dynamic factors according to altitude, their effect from the surface limited by radius r = 1 (300 millibar) is reduced twofold compared to the effect of these factors from the 900 milliba surface which is limited by the same radius.

Function M_2 (1, r, η) (Fig. 7) gives an idea of the effec of thermal factors $-\frac{g}{l}$ (T, z) $+ \frac{\varepsilon}{c_p}$ or, to be more precise, of the effect of local heat flux in the atmosphere and the change of surface pressure. The negative values of function M_2 (1, r, η) in the surrounding area of the atmosphere show that at any level of the atmosphere in this area the local heat flux causes a pressure drop at the earth, while for heat emission ~~and temperature~~ PRESSURE increases.

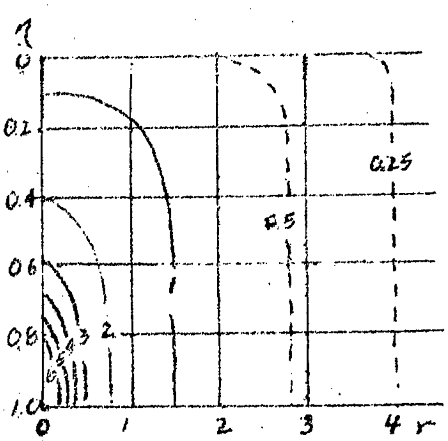

Fig. 5. Field of Function M_1 (ζ , r, η) with ζ = 1.0

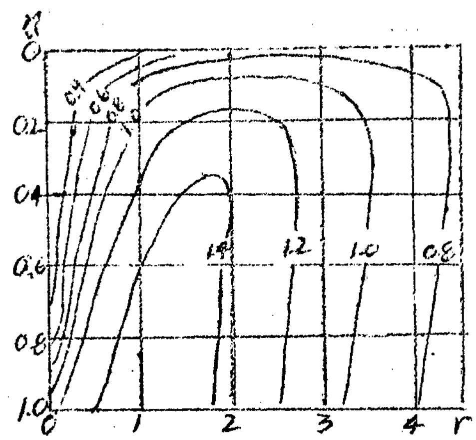

Fig. 6. Field of Function rM_1 (1, r, η)

Compared to the action area of dynamic factors, the regi of activity of thermal factors appears to be more limited. Indeed, the maximum value of function M_2 (1, r, η) along the vertical line with r = 1 is more than twenty times smaller than the maximum value of M_2 (1, r, η) along the verical lir with $r = 0,2$, during the same period in which, for instance, the function of influence M_1 $(1, r, \eta)$ decreases 4 times.

The difference between the areas of influence of thermic and dynamic factors can be recognized by the asymptotic reaction of functions M_2 $(1, r, \eta)$ and M_1 $(1, r, \eta)$. Function I (η, r) which appears as the main part of function M_2 $(1, r, \eta)$, tends toward zero, with higher r values, as $\frac{1}{r^2} e^{-\frac{r}{2}}$, but M_1 $(1, r, \eta)$ shows the same reaction as $\frac{1}{r} e^{-\frac{r}{2}}$.

We also want to show function r M_2 $(1, r, \eta)$ which represen[t] the summary influence of thermal factors on the circumference of radius r, as well as the change of surface pressure. The analysis of function r M_2 $(1, r, \eta)$ shows that the influence of thermal factors in the upper layers of the atmosphere on the change of surface pressure, is usually very small. Inasmu[ch] as the average values of the thermal factors $\frac{g}{l}$ $(T, z) + \frac{\varepsilon}{c_p}$ on circumference r with a sufficiently high r, can be considered important only in the case of greater thermobarometric disturbances in the atmosphere. On the basis of the graph shown in Fig. 8 it can be maintained that the relatively important influence of thermic factors of the upper layers on the chang[e] of surface pressure can exist only in the case of great therm[o] barometric disturbances.

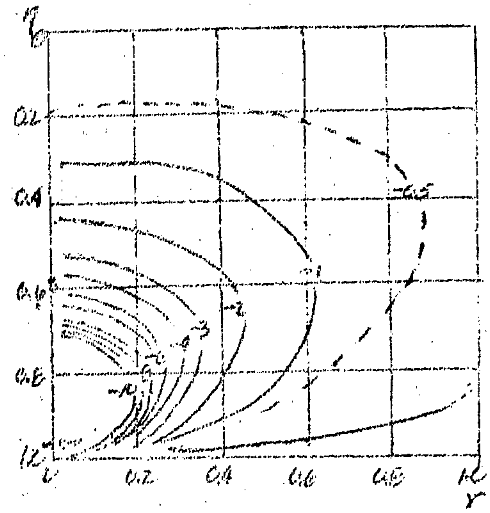

Fig. 7. Field of Function M_2 (ζ, r, η) with ζ=1.0

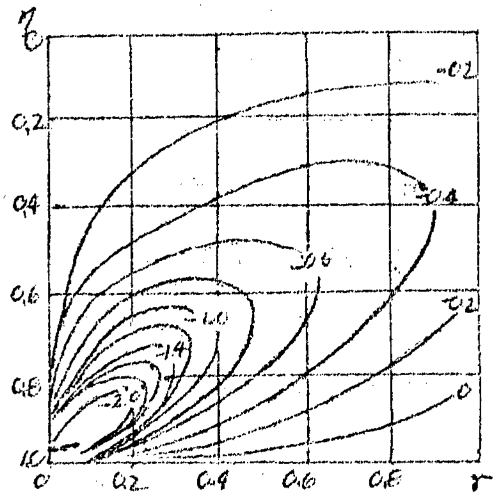

Fig. 8. Field of Function rM_2 (1, r, η)

Now we examine the nature of the functions of influence of M_1 (ζ, r, η) and M_2 (ζ, r, η) with $\zeta < 1$, i.e. the effec of the above mentioned dynamic and thermic factors on the change of pressure at a certain level above the surface of th earth.

Fig. 9 and 10 show functions M_1 (0.7, r, η) and M_1(0.5, r, η). In principle, they do not differ from functions which are similar to them, i.e. from functions for ζ = 1, which were studied above. The difference is only that the area of maximum values of the function differed here from levels ζ = 0.7 and ζ = 0.5, that is to say from levels of examined points.

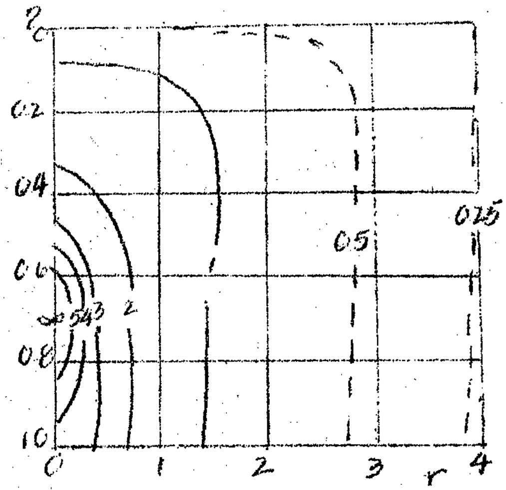

Fig. 9. Field of Function M_1 (ζ , r, η) with ζ = 0.7

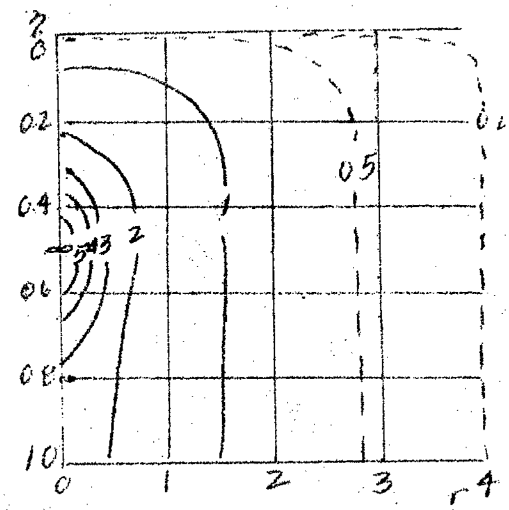

Fig. 10. Field of Function M_1 (ζ , r, η) with ζ = 0.5

A completely different type is function M_2 (ζ , r, η) with $\zeta < 1$ compared to M_2 (1, r, η). Fig. 11 and 12 show functions M_2 (0.7, r, η) and M_2 (0.5, r, η). We can see tha the local flux of heat in the upper part of the atmosphere relative to level ζ causes a drop of temperature; on the other hand, the heat flux in the lower part relative to lev ζ leads to a rise in temperature. The EMISSION of heat a different levels ζ of the atmosphere causes the reverse effect.

At a certain level ζ^* the influences of thermic factor of the upper and lower layers on the pressure change compen sate each other so that the pressure change depends only upon the dynamic factors. This level will be called "mean level" of the atmosphere.

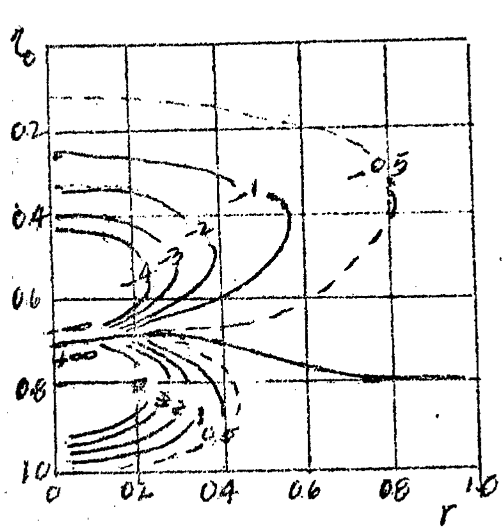

Fig. 11. Field of Function M_2 (ς, r, η) with ς = 0.7

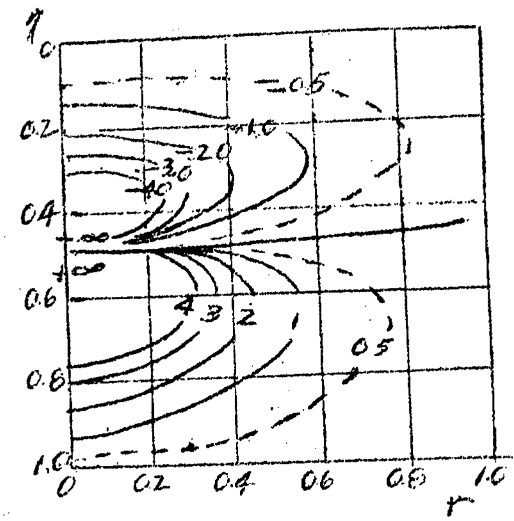

Fig. 12. Field of Function M_2 (ς, r, η) with ς = 0.5

Theoretically, the existence of such a level was alread mentioned in I.A. Kibel's (2) analysis. In the first place the isobaric field at this level, as it was shown by I.A. Kibel, determines the motion of barometric and thermal disturbances in the vicinity of the earth. However, empiricall the existence of the steering current was shown already at an earlier date by S.I. Troitzky (5).

In each concrete synoptic situation function M_2 (ς, r, permits us to determine the position of such a level. In ge ral, this level does not remain constant neither relative t the vast spaces nor the time, because in each practical ca its position will be dependent upon the distribution of heat flux $E = c_p \frac{g}{\ell} (T, z) + \mathcal{E}$.

If we admit that the local heat flux is constant relative to the vertical line along the entire thickness of the atmosphere, the mean level, in this case, would be somewhere between the surfaces of 700 and 500 millibar.

The position of the mean level remains dependent upon the values of thermic and barometric distribution in the atmosphere. With increasing r values the negative values of function $rM_2(\zeta, r, \eta)$ in area $\eta < \zeta$ disappear at a slower rate than the positive values in area $\eta < \zeta$; the conclusion is that with the same relative distribution of the heat flux $E(r, \phi, \eta)$ along the vertical line, the mean level will be higher in the case of great disturbances than in the case of small disturbances.

With $\zeta \rightarrow 0$ solution (2.44) for $\frac{\partial z}{\partial t}$ is converted to the following expression:

$$\left.\frac{\partial z}{\partial t}\right|_{\zeta=0} = \frac{1}{2\pi} \int_0^{\infty} \int_0^{2\pi} m^2 \left[\frac{g}{l}(z, \Delta z) - \beta \frac{\partial z}{\partial x}\right]_{\zeta=0} \frac{U(1,r)}{2} r d\phi dr ;$$

$$+ \frac{1}{2\pi} \int_0^{\infty} \int_0^{2} \frac{R}{g} \left[\frac{g}{l}(T, z) + \frac{\varepsilon}{c_p}\right]_{\zeta=0} \frac{U'(1, r)}{4} r d\phi dr, \quad (2.$$

where

$$U'(1, r) = U(1,r) - 2I(1, r),$$

i.e.

$$U'(l, r) = \begin{cases} U\ (l, r) & \text{with } r > 0 \\ 0 & \text{with } r = 0. \end{cases} \qquad (2.5\ldots$$

The functions of influence $M_1\ (\zeta, r, \eta)$ and $M_2(\zeta, r, \eta)$ characterize the region of dependence of the studied value $\frac{\partial z}{\partial t}$ at the respective point upon the environmental setting of the fields of meteorological elements; these functions ca[n] also be interpreted on the basis of the principle of reversability in the following manner.

At a certain point of space (x, y, ζ) we place a single "source of substance".

$$B = m^2 \left[\frac{g}{l}\ (z, \Delta z) + \beta \frac{\partial z}{\partial x}\right] \quad \text{or} \quad A = \frac{R}{g} \left[\frac{g}{l}\ (T, z) + \frac{\mathcal{E}}{c_p}\right].$$

Accordingly, function $M_1\ (\eta, r, \zeta)$ or $M_2\ (\eta, r, \zeta)$ causes the change of pressure in the surrounding space throu[gh] the influence of this source. The positive source of the dynamic substance B at the level results in an increase of pre[s]sure in the surrounding area at all levels, reflected by spa[ce] function $M_1(\eta, r, \zeta)$. It can be easily seen that function $M_1\ (\eta, r, \zeta)$ is identical with $M_1\ (\zeta, r, \eta)$.

The single "thermal source A" at level ζ results in decreased pressure in the upper part of the atmosphere according to function $M_2\ (\eta, r, \zeta)$ as far as level ζ is concer[n] as well as a rise of pressure in the lower part of the atmosphere. Negative sources of the above mentioned dynamic and thermic substances cause, in an analogous manner, a reverse tendency.

Such an interpretation of the influence function gives us a clear idea of the radius of influence of thermic-barometric disturbances in the atmosphere on the change of pressur changes in neighboring regions.

An analogous interpretation will also be given by the solutions for $\frac{\partial T}{\partial t}$ and τ, which will be described in the following paragraphs.

3. Equations for Temperature Changes.

The equations for temperature changes at different levels will be obtained through the separation from the equation system (2.1) - (2.3) of the vertical velocity τ and derivative $\frac{\partial z}{\partial t}$.

To this end we differentiate (2.1) with ζ. We obtain:

$$\frac{\partial}{\partial \zeta}\left(\Delta \frac{\partial z}{\partial t}\right) + \frac{\partial}{\partial \zeta}\left[\frac{g}{l}(z, \Delta z) + \beta \frac{\partial z}{\partial x}\right] = \frac{l^2}{Pg}\frac{\partial^2}{\partial \zeta^2} . \tag{3.1}$$

The equation of statics (2.3) results as follows:

$$\frac{\partial}{\partial \zeta}\left(\Delta \frac{\partial z}{\partial t}\right) = - \frac{R}{g}\frac{1}{\zeta}\Delta\frac{\partial T}{\partial t} . \tag{3.2}$$

$$\frac{\partial}{\partial \zeta}\left[\frac{g}{l}(z, \Delta z) + \beta \frac{\partial z}{\partial \chi}\right] = -\frac{R}{g}\,\frac{1}{\zeta}\left[\frac{g}{l}(T, \Delta z) + \frac{g}{e}(z, \Delta T) - \right.$$

$$\left. + \beta \frac{\partial T}{\partial \chi}\right]. \qquad (3.3$$

We incorporate (3.2) and (3.3) into (3.1) and obtain:

$$-\frac{1}{\zeta}\Delta \frac{\partial T}{\partial t} - \frac{1}{\zeta}\left[\frac{g}{l}(T, \Delta z) + \frac{g}{e}(z, \Delta T) + \beta \frac{\partial T}{\partial \chi}\right] = \frac{l^2}{PR}\frac{\partial^2 \tau}{\partial \zeta^2}. \qquad (3.4$$

To exclude $\frac{\partial^2 \tau}{\partial \zeta^2}$ from (3.4) we dissolve the equation in regard to τ and differentiate same twice with ζ assuming, as previously, that parameter m^2 changes according to altitude. We obtain:

$$\left(\zeta \frac{\partial^2}{\partial \zeta^2} + 2\frac{\partial}{\partial \zeta}\right)\frac{\partial T}{\partial t} - \left(\zeta \frac{\partial^2}{\partial \zeta^2} + 2\frac{\partial}{\partial \zeta}\right)\left[\frac{g}{l}(T, z) + \frac{\varepsilon}{c_p}\right] =$$

$$= \frac{m^2 l^2}{PR}\frac{\partial^2 \tau}{\partial \zeta^2}. \qquad (3.5$$

Now we multiply (3.4) with m^2 and subtract from (3.5). The result is as follows:

$$\left(\frac{\partial}{\partial \zeta}\zeta^2 \frac{\partial}{\partial \zeta} + m^2 \Delta\right)\frac{\partial T}{\partial t} = f_2(\chi, y, \zeta), \qquad (3.$$

$$f_2(x, y, \zeta) = \frac{\partial}{\partial \zeta} \zeta^2 \frac{\partial}{\partial \zeta} \left[\frac{g}{l} (T, z) + \frac{\varepsilon}{c_p} \right] -$$

$$- m^2 \left[\frac{g}{l} (T, \Delta z) + \frac{g}{l} (z, \Delta T) + \beta \frac{\partial T}{\partial x} \right] . \qquad (3.7)$$

In regard to the nature of limit conditions for equation (3.6) we assume the following (see conditions (2.9) and (2.14

$$\left. \begin{aligned} \left. \frac{\partial T}{\partial t} \right|_{\zeta = 1} &= Q_o (x, y), \\ \left. \zeta \frac{\partial T}{\partial t} \right|_{\zeta = o} &= 0, \end{aligned} \right\} \qquad (3.8)$$

$$Q_o(x, y) = \frac{g}{l} (T_o, z_o) + \frac{\varepsilon_o}{c_p} . \qquad (3.9)$$

The term $(\chi_a - \gamma) \frac{\partial z_o}{\partial t}$ contained in $Q_o(x, y)$ is disregarded in this analysis because this term is of a higher order [1].

In view of the fact that equation (3.6) has a structure which is analogous to the structure of equation (2.6) for $\frac{\partial z}{\partial t}$

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[1] The exclusion of term $(\chi_a - \gamma) \frac{\partial z_o}{\partial t}$ contained in $Q(x, y)$, is not of basic nature. It could also be retained as $\frac{\partial z_o}{\partial t}$ can be determined according to formula (2.44) in the previous paragraph.

the solution for $\frac{\partial T}{\partial t}$ as well as for $\frac{\partial z}{\partial t}$ will be determined by the mean value of exact functions on the peripheries with the center being at the examined point. Therefore, we apply a simpler mathematical method for the solution of equation (3.6). The possibility of applying this method was already mentioned above.

We note equation (3.6) in the cylindrical coordinate system (r, ϕ, ζ), $r = \frac{\sqrt{x^2+y^2}}{m}$, ϕ - polar angle, $\zeta = \frac{p}{P}$ - mentioned altitude. We integrate this equation with φ from zero to 2π.

The result is as follows:

$$\left(\frac{\partial}{\partial \zeta} \zeta^2 \frac{\partial}{\partial \zeta} + \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r}\right) \frac{\partial T}{\partial t} = f_2(r, \zeta), \tag{3.10}$$

$$\frac{\partial T}{\partial t} = \frac{1}{2\pi} \int_0^{2\pi} \frac{\partial T}{\partial t} d\phi, \quad f_2(r, \zeta) = \frac{1}{2\pi} \int_0^{2\pi} f_2(r, \phi, \zeta) d\phi \tag{3.11}$$

These are values $\frac{\partial T}{\partial t}$ and f_2 ascertained on the circumference of radius r.

The limit conditions (3.8) are reflected in the following manner:

$$\left.\begin{aligned} \frac{\partial T}{\partial t}\bigg|_{\rho=1} &= Q_o(r), \\ \zeta\frac{\partial T}{\partial t}\bigg|_{\rho=o} &= 0. \end{aligned}\right\} \tag{3.12}$$

The solution for $\frac{\partial T}{\partial t}$ will be tried in the form of the Fourie-Bessel integral

$$\frac{\partial T}{\partial t} = \int_0^\infty S(\rho, \zeta) J_o(r\rho)\, \rho\, d\rho. \tag{3.13}$$

We assume that function $f_2(r, \zeta)$ and $Q_o(r)$ can be presented in the form of the Fourie-Bessel integral:

$$\left.\begin{aligned} f_2(r, \zeta) &= \int_0^\infty J_o(r\rho)\, \rho\, d\rho \int_0^\infty f_2(r', \zeta) J_o(\rho r')\, r' dr', \\ Q_o(r) &= \int_0^\infty J_o(r\rho)\, \rho\, d\rho \int_0^\infty Q_o(r') J_o(\rho r')\, r' dr'. \end{aligned}\right\} \tag{3.14}$$

We take (3.13) and (3.14) as a basis for equation (3.10) and limit conditions (3.12) and use the known correlation

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial J_o(\rho r)}{dr}\right\} = -\rho^2 J_o(\rho r),$$

In this manner we obtain the following equation and limit co ditions for $S(\rho,\zeta)$:

$$\frac{\partial}{\partial\zeta}\zeta^2\frac{\partial S}{\partial\zeta} - \rho^2 S = F_2(\rho,\zeta), \qquad (3.1$$

$$\left.\begin{aligned} S\quad &= G_2(\rho), \\ \zeta S\big|_{\zeta=0} &= 0, \end{aligned}\right\} \qquad (3.1$$

$$\left.\begin{aligned} F_2(\rho,\zeta) &= \int_0^\infty f_2(r',\zeta)\, J_o(\rho r')r'dr', \\ G_2(\rho) &= \int_0^\infty Q_o(r')J_o(\rho r')\, r'dr'. \end{aligned}\right\} \qquad (3.1$$

In view of the fact that equation (3.15), as far as its structure is concerned, is analogous to equation (2.19) examined in the last paragraph, the general solution of equatio (3.15) will have a form which is analogous to (2.29).

$$S(\rho,\zeta) = C_1\zeta^{\nu_1} + C_2\zeta^{\nu_2} + \frac{1}{\nu_1-\nu_2}\int_0^\zeta F_2(\rho,\zeta)\left[\zeta^{\nu_1}\eta^{\nu_2} - \zeta^{\nu_2}\eta^{\nu_1}\right]d\eta \qquad (3.1$$

$$\nu_1 = -\frac{1}{2} + \sqrt{\frac{1}{4} + \rho^2}, \quad \nu_2 = -\frac{1}{2} - \sqrt{\frac{1}{4} + \rho^2}.$$

We take arbitrarily constant quantities C_1 and C_2 from th limit conditions and obtain:

$$S(\rho, \zeta) = -\frac{1}{2\mu}\int_0^1 F_2(\rho,\eta)\frac{1}{\sqrt{\zeta\eta}}\left[\delta_1\left(\frac{\eta}{\zeta}\right)^{\mu} + \delta_2\left(\frac{\zeta}{\eta}\right)^{\mu} - (\zeta\eta)^{\mu}\right]d\eta$$

$$+ G(\rho)\,\zeta^{\mu - \frac{1}{2}}, \qquad (3.19$$

$$\mu = \sqrt{\frac{1}{4} + \rho^2}.$$

We take (3.19) and (3.13) as a basis and replace $F_2(\rho, \eta$ and G (ρ) according to formula (3.17). If we also change the order of integration we obtain:

$$\frac{\partial T}{\partial t} = -\int_0^1\int_0^{\infty} f_2(r', \eta)\left\{\frac{1}{2\sqrt{\zeta\eta}}\int_0^{\infty}\left[\delta_1\left(\frac{\eta}{\zeta}\right)^{\mu} + \right.\right.$$

$$\left.\left. + \delta_2\left(\frac{\zeta}{\eta}\right)^{\mu} - (\zeta\eta)^{\mu}\right]\frac{J_o(r\rho)J_o(r'\rho)}{\mu}\right\} r'dr'd\eta +$$

$$+ \int_0^{\infty} Q_o(r')\left\{\frac{1}{\sqrt{\zeta}}\int_0^{\infty}\zeta^{\mu} J_o(r\rho)\,J_o(r'\rho)\rho\,d\rho\right\} r'dr'. \quad (3.$$

Now we assume that in (3.20) $r = 0$. As a result we obtain the solution for $\frac{\partial T}{\partial t}$ at the points which are located at vario levels ζ, along the coordinate axis.

$$\frac{\partial T}{\partial t} = - \int_0^1 \int_0^\infty f_2(r_1, \eta) M_3(\zeta, r', \eta) r' dr' d\eta + \int_0^\infty Q_o(r') I(\zeta, r') r' dr', \qquad (3.2$$

$$M_3(\zeta, r', \eta) = \frac{1}{2\sqrt{\zeta\eta}} \int_0^\infty \left[\delta_1 \left(\frac{\eta}{\zeta}\right)^\mu + \delta_2 \left(\frac{\zeta}{\eta}\right)^\mu - (\zeta\eta)^\mu\right] \frac{J_o(r'\rho)}{\mu} \rho d\rho \qquad (3.2$$

and

$$I(\zeta, r') = \int_0^\infty \zeta^{\mu - \frac{1}{2}} J_o(r'\rho) \rho d\rho \qquad (3.2$$

These are functions which had already been used previously. We take $f_2(r', \eta)$ from (3.11) and (3.7) as a basis for (3.21) and partially integrate thermic factor $\frac{g}{l}(T, z) + \frac{\varepsilon}{c_p}$

Thus we can finally give a description of solution (3.21) for $\frac{\partial T}{\partial t}$ as follows [1] :

---

[1] Derivative $\frac{\partial T}{\partial t}$ can be directly obtained from solution (2.3 for $\frac{\partial z}{\partial t}$ by differentiating with ζ and utilizing correlatio

$$\frac{\partial T}{\partial t} = \frac{1}{2\pi}\int_0^\infty \int_0^{2\pi} \left[\frac{g}{l}(T_o, z_o) + \frac{\mathcal{E}_o}{c_p}\right] I(\zeta, r) r d\varphi\, dr +$$

$$+ \frac{1}{2\pi}\int_0^1 \int_0^\infty \int_0^{2\pi} \eta \frac{\partial}{\partial \eta}\left[\frac{g}{l}(T, z) + \frac{\mathcal{E}}{c_p}\right] M_4(\zeta, r, \eta)\, r d\varphi\, dr d\eta +$$

$$+ \frac{1}{2\pi}\int_0^1 \int_0^\infty \int_0^{2\pi} m^2 \left[\frac{g}{l}(T, \Delta z) + \frac{g}{l}(z, \Delta T) + \beta\frac{\partial T}{\partial \chi}\right] M_3(\zeta, r, \eta)$$

$$r d\varphi\, dr d\eta , \qquad (3.2$$

$$M_4(\zeta, r, \eta) = \eta \frac{\partial M_3}{\partial \eta} .$$

(continued) $$\frac{\partial T}{\partial t} = -\frac{g}{R}\zeta\frac{\partial}{\partial \zeta}\left(\frac{\partial z}{\partial t}\right) .$$

Then we obtain:

$$\frac{\partial T}{\partial t} = \frac{1}{2\pi}\int_0^\infty \int_0^{2\pi} Q_o(r,\varphi) G_1(\zeta, r) r d\varphi dr + \frac{1}{2\pi}\int_0^1 \int_0^\infty \int_0^{2\pi} f(r,\varphi,\eta)$$

$$G_2(\zeta, r, \eta) r d\varphi dr d\eta ,$$

$$f(r,\varphi,\eta) = \frac{gm^2}{R}\left[\frac{g}{l}(z, \Delta z) + \beta\frac{\partial z}{\partial \eta}\right] + \frac{\partial}{\partial \eta}\eta\left[\frac{g}{l}(T, z) + \frac{\mathcal{E}}{c_p}\right.$$

The functions of influence M_3 and M_4 are expressed by functions of type $U(x, r)$ and $I(x, r)$ which were used in the preceding paragraph.

$$\left.\begin{aligned} M_3(\zeta,r,\eta) &= \frac{1}{2}\left[\frac{\delta_1}{\zeta} U\left(\frac{\eta}{\zeta}, r\right) + \frac{\delta_2}{\eta} U\left(\frac{\zeta}{\eta}, r\right) - U(\zeta\eta, r)\right], \\ M_4(\zeta,r,\eta) &= \frac{1}{2}\left\{\frac{\delta_1}{\zeta} I\left(\frac{\eta}{\zeta}, r\right) - \frac{\delta_2}{\eta} I\left(\frac{\zeta}{\eta}, r\right) - I(\zeta\eta, r)\right\} - \\ &\quad - \frac{1}{2} M_3(\zeta,r,\eta). \end{aligned}\right\} (3.2$$

Fig. 13-17 show the function of $I(\zeta,r)$ and the sections in the vertical field value ζ of functions $M_4(\zeta,r,\eta)$ and $M_3(\zeta,r,\eta)$.

The analysis of the values of function $I(\zeta,r)$ (Fig. 13) shows that the first term on the right (3.24) calculated according to altitude, steadily vanishes with the vaning ζ .

(continued)

$$G_2(\zeta,r,\eta) = -\zeta\frac{\partial M_1}{\partial\zeta} = \frac{1}{2}\left\{\frac{\delta_1}{\zeta} I\left(\frac{\eta}{\zeta}, r\right) - \frac{\delta_2}{\eta} I\left(\frac{\zeta}{\eta}, r\right) + I(\zeta\eta, r)\right\}$$

$$+ \frac{-1}{2} M_3(\zeta,r,\eta) + \alpha M^*(\zeta\eta, r),$$

$$G_1(\zeta,r) = \zeta\frac{\partial M^*(\zeta,r)}{\partial\zeta} = I(\zeta,r) - \alpha M^*(\zeta,r).$$

Function $M_4(\zeta,r,\eta)$ (Fig. 14 and 15) is used for calculating the irregular distribution of the local heat flux along the vertical. The first two integrals on the right (3.24) give complete data on the influence of local heat flux distributed over the entire atmosphere and on the temperature change at a fixed point.

Analytically, function $M_4(\zeta,r,\eta)$ consists of two parts:

$$M_4(\zeta,r,\eta) = X(\zeta,r,\eta) - \frac{1}{2} M_3(\zeta,r,\eta), \quad (3.26$$

$$X(\zeta,r,\eta) = \frac{1}{2}\left\{\frac{\delta_1}{\zeta} I\left(\frac{\eta}{\zeta}, r\right) - \frac{\delta_2}{\eta} I\left(\frac{\zeta}{\eta}, r\right) - I(\zeta\eta,r)\right\}. \quad (3.2$$

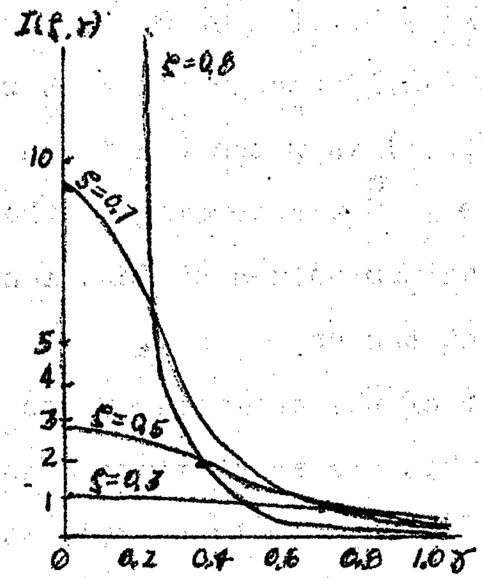

Fig. 13. Graph of Function $I(\zeta,r)$ for various ζ.

In the environment of radius $r \approx 0.5$ $X(\zeta,r,\eta)$ forms the main part of values $M_4(\zeta,r,\eta)$. On the other hand, $\frac{1}{2} M_3(\zeta,r$ appears to be relatively small. With higher values of $r (r \geqslant 1)$ the corresponding equilibrium of both components contained in $M_4(\zeta,r,\eta)$, is outbalanced; but each of these components is small.

Function $M_3(\zeta,r,\eta)$ (Fig. 16 and 17) gives an idea of the sphere of influence of the dynamic factors $m_2\left[\frac{g}{l}(T,\Delta z) + \frac{g}{l}(z,\Delta T) + \beta \frac{\partial T}{\partial \chi}\right]$ on the change of temperature at the respective point on level ζ. This function has its maximum values in the immediate vicinity of the examined point and vanishes with the increasing r but also in the direction of the upper and lower boundaries of the atmosphere. $\eta = 0$ and $\eta = 1$ $M_3(\zeta,r,\eta) = 0$, if $\zeta \neq 0$.

It can be easily recognized that with higher r values function $M_3(\zeta,r,\eta)$ is presented in an asymptotic manner as follows: $\frac{1}{r^2} e^{-\frac{r}{2}}$. According to the increase of r values, the area of maximum values of function $M_3(\zeta,r,\eta)$ with level ζ is formed at the upper part.

The result of the analysis of the properties of function $M_3(\zeta,r,\eta)$ is that the temperature change at any level ζ expressed by the last term in (3.24), is determined by the dynamic factors of the atmospheric layers bordering level ζ to a greater extent. In regard to level ζ the dynamic factors of the upper layers have a higher ~~[illegible]~~ weight than similar factors in lower layers.

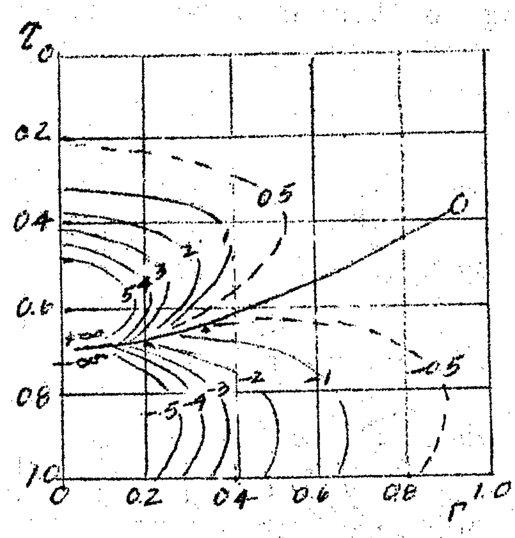

Fig. 14. Field of Function $M_4(\zeta, r, \eta)$ with $\zeta = 0.7$

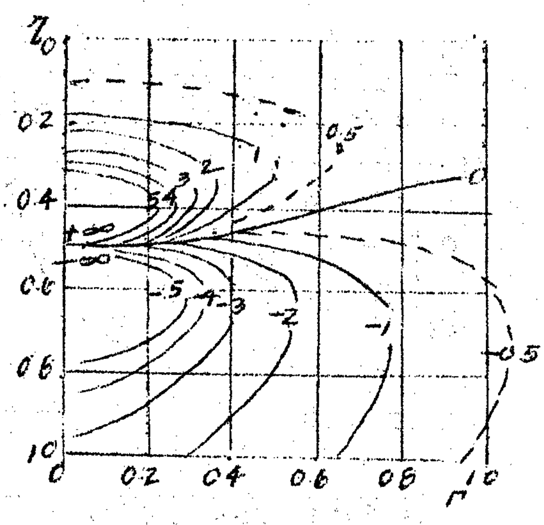

Fig. 15. Field of Function $M_4(\zeta, r, \eta)$ with $\zeta = 0.5$

Now we assume in what manner the solution for $\frac{\partial T}{\partial t}$ describes the change of temperature at the upper border of the atmosphere.

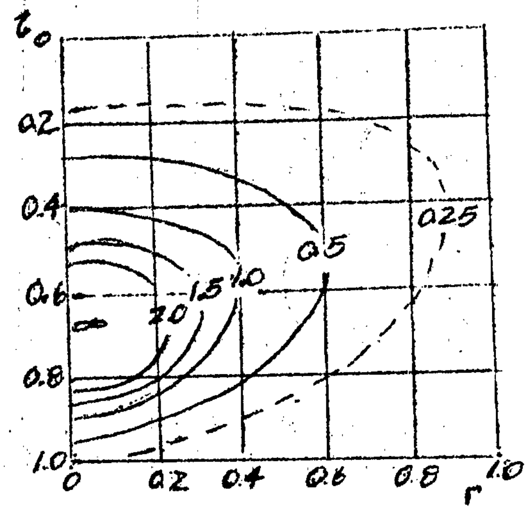

Fig. 16. Field of Function $M_3(\zeta, r, \eta)$ with $\zeta = 0.7$

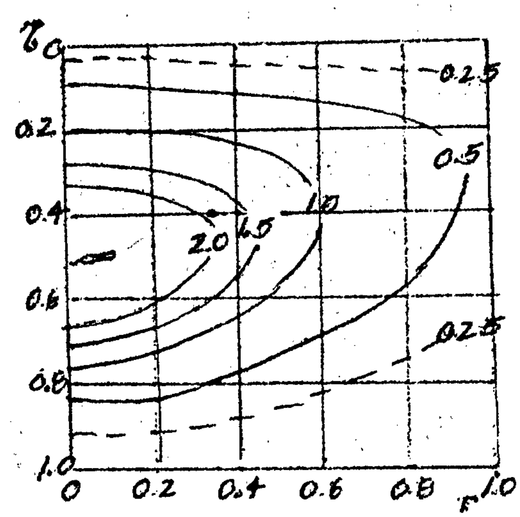

Fig. 17. Field of Function $M_3(\zeta, r, \eta)$ with $\zeta = 0.5$

With $\zeta \to 0$ function $I(\zeta, r) \to 0$, $M_4(\zeta, r, \eta)$ is converted to $\frac{\delta_1}{2\zeta}\left[I(1, r) - \frac{1}{2}U(1, r)\right]$, $M_3(\zeta, r, \eta)$ and $\frac{\delta_1}{2\zeta}U(1, r)$. Then solution (3.24) takes the following shape with $\zeta \to 0$:

$$\left.\frac{\partial T}{\partial t}\right|_{\zeta=0} = -\frac{1}{2\pi}\int_0^\infty \int_0^{2\pi} \eta \frac{\partial}{\partial \eta}\left[\frac{g}{l}(T, z) + \frac{\varepsilon}{c_p}\right]_{\zeta=0} \frac{U'(1,r)}{4} r d\varphi dr +$$

$$+ \frac{1}{2\pi}\int_0^\infty \int_0^{2\pi} m^2 \left[\frac{g}{l}(T, \Delta z) + \frac{g}{l}(z, \Delta T) + \beta\frac{\partial T}{\partial x}\right]_{\zeta=0} \frac{U(1,r)}{2} r d\varphi dr, \tag{3.28}$$

Analogous to (2.56)

$$U'(1,r) = \begin{cases} U(1,r) & \text{with } r > 0, \\ 0 & \text{with } r = 0. \end{cases}$$

Now we recall correlation (3.3)

$$\zeta\frac{\partial}{\partial\zeta}\left[\frac{g}{l}(z, \Delta z) + \beta\frac{\partial z}{\partial x}\right] = -\frac{R}{g}\left[\frac{g}{l}(T, \Delta z) + \frac{g}{l}(z, \Delta T) + \beta\frac{\partial T}{\partial x}\right]$$

Assuming that

$$\lim_{\zeta\to 0}\left\{\zeta\frac{\partial}{\partial\zeta}\left[\frac{g}{l}(z, \Delta z) + \beta\frac{\partial z}{\partial x}\right]\right\} = 0\,^1, \tag{3.29}$$

---

[1] Qualification (3.29) means that the kinetic energy of the u of mass $\frac{V^2}{2}$ according to altitude does not increase faster t the increase of $\frac{1}{\zeta}$.

we obtain

$$\left[\frac{g}{l}(T,\Delta z) + \frac{g}{l}(z,\Delta T) + \beta \frac{\partial T}{\partial x}\right]_{\zeta=0} = 0. \qquad (3.30$$

The result of (3.30) is that derivatives $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial y}$ at the upper border of the atmosphere equal zero; this means tha the temperature on the surface ζ= const, is constant with $\zeta \to 0$. In other words,

$$(T,\ z)_{\zeta=0} = 0. \qquad (3.3$$

We further assume that $\zeta \to 0$

$$\zeta \frac{\partial \varepsilon}{\partial \zeta}\bigg|_{\zeta=0} = 0. \qquad (3.3$$

Meeting conditions (3.30) to (3.32) from (3.28) we obtain th result:

$$\frac{\partial T}{\partial t}\bigg|_{\zeta=0} = 0. \qquad (3.3$$

4. Equations for Vertical Velocity.

Vertical motion appears as a component of the mechanism of atmospheric circulation. The redistribution of kinetic, p tential and internal energies of air from one level to the other is realized through vertical currents. Therefore, in the study of atmospheric processes the examination of vertical motion should be given careful attention.

At the same time, vertical motions are of interest as suc because they are the main factor in the process of formation of cloud conditions and precipitation.

To obtain an equation for vertical motion ζ in the atmosphere, it is indispensible to exclude derivatives $\frac{\partial z}{\partial t}$ and $\frac{\partial T}{\partial t}$ from system (2.1) and (2.3).

For this purpose we differentiate the equation (2.1) with ζ

We obtain:

$$\frac{l^2}{Pg}\frac{\partial^2 \tau}{\partial \zeta^2} = \frac{\partial}{\partial \zeta}\left(\Delta \frac{\partial z}{\partial t}\right) + \frac{\partial}{\partial \zeta}\left[\frac{g}{l}(z, \Delta z) + \beta \frac{\partial z}{\partial x}\right]. \quad (4$$

The equation of statics gives us correlations

$$\frac{\partial}{\partial \zeta}\left(\Delta \frac{\partial z}{\partial t}\right) = -\frac{R}{g}\frac{1}{\zeta}\Delta \frac{\partial T}{\partial t}, \quad (4.2$$

$$\frac{\partial}{\partial \zeta}\left[\frac{g}{l}(z, \Delta z) + \beta \frac{\partial z}{\partial x}\right] = -\frac{R}{g}\frac{1}{\zeta}\left[\frac{g}{l}(T, \Delta z) + \frac{g}{l}(z, \Delta T) + \right.$$

$$\left. + \beta \frac{\partial T}{\partial x}\right]. \quad (4.3$$

We take (4.2) and (4.3) as a basis for (4.1). Then we obtain

$$\frac{l^2}{PR}\frac{\partial^2 \tau}{\partial \zeta^2} = -\frac{1}{\zeta}\Delta\frac{\partial T}{\partial t} - \frac{1}{\zeta}\left[\frac{g}{l}(T,\Delta z) + \frac{g}{l}(z,\Delta T) + \beta\frac{\partial T}{\partial x}\right]. \quad (4.4)$$

We exclude the derivative $\frac{\partial T}{\partial t}$ of (4.4) through the equation of heat flux (2.2) and obtain:

$$\zeta^2\frac{\partial^2 \tau}{\partial \zeta^2} + \frac{R^2}{gl^2}\Delta\left[T(\gamma_a - \gamma)\tau\right] = \zeta f_3(x,y,\zeta), \quad (4.5)$$

$$f_3(x,y,\zeta) = -P\frac{R}{l^2}\left\{\frac{g}{l}\Delta(T,z) + \frac{\Delta\varepsilon}{c_p} + \frac{g}{l}(T,\Delta z) + \frac{g}{l}(z,\Delta T) + \beta\frac{\partial T}{\partial x}\right\}. \quad (4.6)$$

We assume that the changes of value $T(\gamma_a - \gamma)$ in regard the vertical and horizontal are small compared to the relative changes of vertical velocity. Thus equation (4.5) can formulated in the following manner:

$$\zeta^2\frac{\partial^2 \tau}{\partial \zeta^2} + m^2\Delta\tau = \zeta f_3(x,y,\zeta), \quad (4.7)$$

m^2 - parameter introduced above.

The limit conditions for equation (4.7) are as follows:

$$\tau\big|_{\zeta=0} = 0, \qquad \tau\big|_{\zeta=1} = \tau_0, \quad (4.8)$$

$$\tau_0 = g\rho_0 \frac{\partial z_0}{\partial t} \quad \text{- small quantity.}$$

We now come to the cylindrical coordinate system r, φ, ζ, already used above.

In this coordinate system equation (4.7) assumes the following shape:

$$\zeta^2 \frac{\partial^2 \tau}{\partial \zeta^2} + \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \tau}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \tau}{\partial \varphi^2} = \zeta f_3(r, \varphi, \zeta). \quad (4.9)$$

Integrating this equation with φ from zero to 2π, we obtain the following equation for function $\bar{\tau} = \frac{1}{2\pi}\int_0^{2\pi} \tau d\varphi$:

$$\zeta^2 \frac{\partial^2 \bar{\tau}}{\partial \zeta^2} + \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \bar{\tau}}{\partial r} = \zeta F_3(r, \zeta), \quad (4.10)$$

$$F_3(r, \zeta) = \frac{1}{2\pi}\int_0^{2\pi} f_3(r, \varphi, \zeta)\, d\varphi. \quad (4.11)$$

The limit conditions of equation (4.10) are as follows:

$$\left.\begin{aligned} \bar{\tau}\big|_{\zeta=0} &= 0, \\ \bar{\tau}\big|_{\zeta=1} &= \bar{\tau}_0(r). \end{aligned}\right\} \quad (4.12)$$

The solution of equation (4.10) is obtained through

$$\bar{\tau}(r, \zeta) = \int_0^{\infty} S(\rho, \zeta)\, J_0(r\rho)\, \rho d\rho. \quad (4.13)$$

As we have done above, we assume that function $F_3(r,\zeta)$ and $\overline{\tau}_o(r)$ can be presented as intervals according to Fourie-Bessel

$$\left.\begin{aligned} F_3(r,\zeta) &= \int_0^\infty J_o(r\rho)\rho\, d\rho \int_0^\infty F_3(r',\zeta)\, J_o(\rho r')r'dr', \\ \overline{\tau}_o(r) &= \int_0^\infty J_o(r\rho)\,\rho\, d\rho \int_0^\infty \overline{\tau}_o(r')\, J_o(\rho r')r'dr' . \end{aligned}\right\} \tag{4.14}$$

We take (4.13) and (4.14) as basis for (4.10) and (4.12). Analogous to the above we obtain the following differential equations and limit conditions for the new unknown function $S(\rho,\zeta)$:

$$\zeta^2 \frac{\partial^2 S}{\partial \zeta^2} - \rho^2 S = \zeta F_3(\rho,\zeta), \tag{4.15}$$

$$\left.\begin{aligned} S\Big|_{\zeta=0} &= 0, \\ S\Big|_{\zeta=1} &= G_3(\rho), \end{aligned}\right\} \tag{4.16}$$

$$
\left.\begin{aligned}
F_3(\rho,\zeta) &= \int_0^\infty F_3(r',\zeta)\, J_o(\rho r')\, r'dr', \\
G_3(\rho) &= \int_0^\infty \tau_o(r')\, J_o(\rho r') r'dr'.
\end{aligned}\right\} \qquad (4.17)
$$

Following is the general solution of the equation (4.15)

$$
S(\rho,\zeta) = C_1 \zeta^{\frac{1}{2}+\mu} + C_2 \zeta^{\frac{1}{2}-\mu} + \frac{1}{2\mu}\int_0^\zeta F_3(\rho,\eta)\sqrt{\frac{\zeta}{\eta}}
$$

$$
\left[\left(\frac{\zeta}{\eta}\right)^\mu - \left(\frac{\eta}{\zeta}\right)^\mu\right] d\eta, \qquad (4.18)
$$

as above

$$
\mu = \sqrt{\frac{1}{4} + \rho^2}.
$$

In the selection of C_1 and C_2 we will see to it that solution (4.18) meets limit conditions (4.16). The result is as follows

$$
S(\rho,\zeta) = -\frac{1}{2\mu}\int_0^1 F_3(\rho,\eta)\sqrt{\frac{\zeta}{\eta}} - \left[\delta_1\left(\frac{\eta}{\zeta}\right)^\mu + \delta_2\left(\frac{\zeta}{\eta}\right)^\mu - \right.
$$

$$
\left. - (\zeta\eta)^\mu\right] d\eta + G_3(\rho)\,\zeta^{\frac{1}{2}+\mu}. \qquad (4.19)
$$

Bearing in mind that $F_3(\rho, \xi)$ and $G_3(\rho)$ can be expressed the integral way (4.17), we will use it as a basis for (4.19 Moreover, the obtained expression is taken as a bases for (4.13). Now we change the order of integration and assume th $r = 0$.

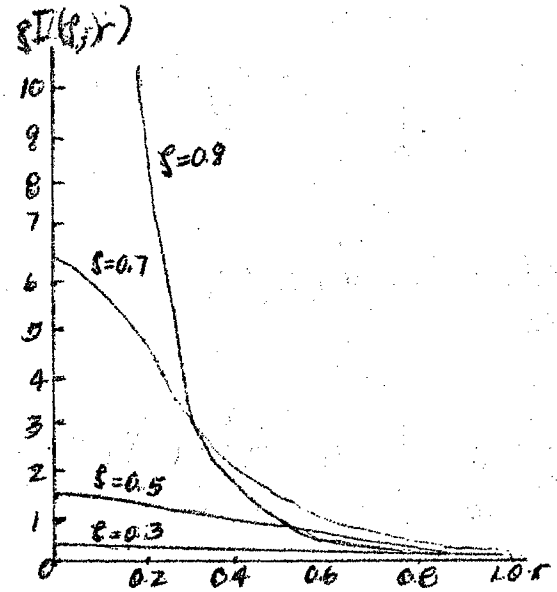

Fig. 18. Graph of Function $\xi I(\xi, r)$ for various ξ.

Then we obtain:

$$\tau(\xi) = -\int_0^1 \int_0^\infty F_3(r', \eta)\, \xi M_3(\xi, r', \eta) r' dr' d\eta +$$

$$+ \int_0^\infty \tau_0(r') \xi I(\xi, r') r' dr', \qquad (4.2$$

$M_3(\xi, r', \eta)$ and $I(\xi, r')$ - Functions (3.22) and (3.23) introduced previously.

Substituting $F_3(r,\eta)$ and $\tau_o(r)$ in (4.20) according to (4.11) and (4.12) through (4.6), we formulate solution (4.20) in the final form as follows:

$$\tau(\xi) = \frac{1}{2\pi}\int_0^1\int_0^\infty\int_0^{2\pi} P \frac{R}{l^2}\left\{\frac{g}{l}\,\Delta(T,z) + (T,\Delta z) + (z,\Delta T)\right.$$

$$\left. + \frac{\Delta\varepsilon}{c_p} + \beta\frac{\partial T}{\partial x}\right\}\xi M_3(\xi, r,\eta)rd\varphi\, drd\eta +$$

$$+ \frac{1}{2\pi}\int_0^\infty\int_0^{2\pi} g\rho_o \frac{\partial z_o}{\partial t}\xi\; I(\xi, r)rd\varphi dr. \qquad (4.2$$

The graphic presentation of function $\xi I(\xi,r)$ with variou ξ (Fig. 18) shows that the second term in the right part (4.2 quickly vanishes at the respective altitude.

Considering that $\tau_o = g\rho_o \frac{\partial z_o}{\partial t}$ usually does not excee(10 millibar (12 hours), we may disregard the second component in (4.21).

Fig. 19 and 20 show the fields of functions $\xi M_3(\xi,r,\eta)$ for different ξ .

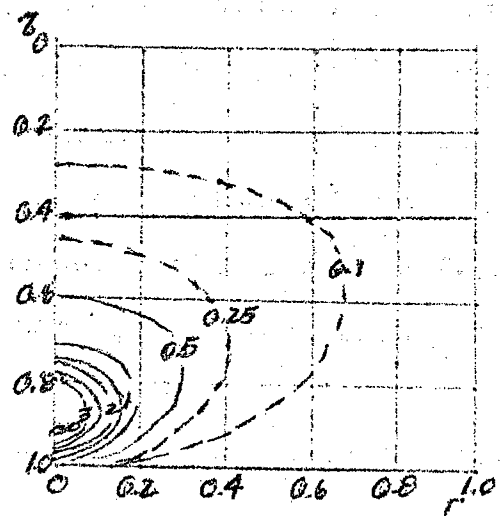

Fig. 19. Field of Function $\zeta M_3(\zeta, r, \eta)$ with $\zeta = 0.9$

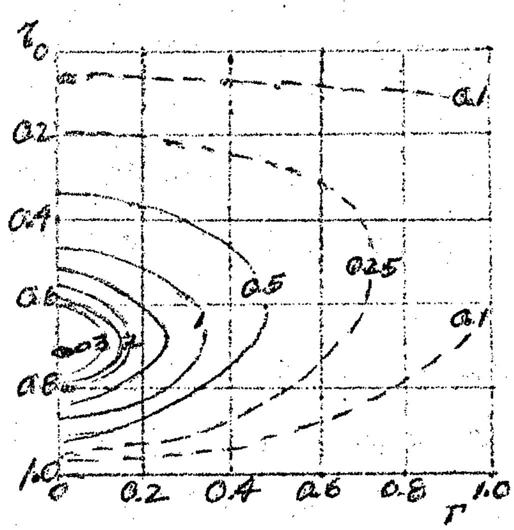

Fig. 20. Field of Function $\zeta M_3(\zeta, r, \eta)$ with $\zeta = 0.7$

The properties of function $M_3(\zeta, r, \eta)$ show that in the formation of vertical motions in the intermediary troposphere the dynamic processes in the intermediary and upper parts of the troposphere play the decisive part. The vertical motions at the level of 3-5 km are mainly determined by particularities of the pressure fields, temperature and heat flux at the 3-8 levels.

Equation (4.21) shows that in the source region of the heat flux the terms of type $\Delta \left[-\frac{g}{l} (T, z) + \frac{\varepsilon}{c_p} \right]$ will always provide the anabatic motions. But in the heat discharge regions they will provide the catabatic motions. Consequently, part of the heat influx entering any important region of the atmosphere is transferred, together with the vertical currents to higher layers.

On the other hand, the heat discharge is partly compensated by the transfer of heat through vertical currents from higher layers.

This points out the important role played by vertical motions of the atmosphere in the redistribution of thermic energy.

Factor $\beta \frac{\partial T}{\partial x}$ in the solution for τ which appears in the initial equations of the parameter change of the coriolis according to latitude, is important only in the presence of great thermic disturbances of the atmosphere. In the eastern part of the thermal crest this factor becomes an anabatic component of vertical velocity, in the western part of the crest however, a catabatic component.

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